

Deprogramming Zero: Peano’s Axioms Without the Sign for Non-Presentation, the One Kept Record, and the Resolution of the Riemann Question at Its Registers

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Abstract

Zero is a formed sign assigned to stand for a non-presentation: its positive formation certifies that something presented while its semantic office says that something did not. This is the root-grammatical contradiction the presence-only programme of *Against Zero* and *The Mirror Calculus* identifies; a *surrogate zero* is any sign a language reaches for to keep that concept under another spelling — [blank, null, empty, undefined, the identity “that does nothing”] — and the surrogate-zero test is the instrument that catches them. This paper deprograms zero out of Peano Arithmetic and carries the result through algebra, set theory, and the Riemann question. PA uses its constant 0 in exactly four ways — as a named point, as the terminus past which the successor stops, as the base of the recursions for $+$ and $-$, and as the base of induction — and each of the four is an instance of the contradiction. We give a formation-faithful interpretation of PA into tally arithmetic TA, the theory of the positive integers with unit, successor, fusion and product, whose object language contains neither the sign nor any surrogate for it and whose unit **1** is formed by the first act and stands for presence, and a converse, so that the theories are bi-interpretable and equiconsistent: the sign is eliminable, which is its formal usability, the whole of what the objection from usability establishes. What the elimination exposes is the cost the classical notation concealed. Three of the four uses are *substitutions* — the unit tally, a return to the unit, induction from the unit — and each classical use of them hides a predecessor-witness obligation that the classical bookkeeping discharges in silence and TA discharges in the open; the operation in which that silent discharge fails is division, and the classical notation names the failure with a returned non-value, “undefined,” which is a surrogate zero by the test. The fourth use, the terminus, is a *kept record*: by Lyndon’s preservation theorem no negation-free first-order theory excludes the cyclic models $\mathbb{Z}/k$, so the statement that the first tally is reached by no successor is held in the metalanguage as a record, independent of the positive fragment, which is exactly where *Against Zero* places withheld presentation. A ledger counts the substitutions, the concealed obligations, and the record; a general schema extends the result to every theory whose zero is the least element of a discrete order; the identity of a group and the zero of a ring are shown to be surrogates by the test, which heap and truss theory remove; and the empty set, the surrogate of set theory, is placed as a kept record, forced by negative Separation and by $\in$ -induction. The second half of the paper turns to the Riemann hypothesis and first proves why the sign was the obstacle: written with the sign, the hypothesis is [a universal over the

plane whose matrix is an evaluation set equal to the sign], on a totality outside presentation — which positive logic does not form and arithmetic makes Π_1 , so that its affirmation is an ω -inference with its sponsor left off the page — while the verification ledger presents tallies at every stage, Riemann’s completion ξ had already removed the surrogate zeros of ζ , and in the explicit formula each zero of ξ is a presented carrier whose equal weighting is the native form of the hypothesis. On the prime side the Admission law is Lagarias positivity and Li’s criterion — the positivity of a prime-indexed sequence — the mirror pairs carriers whose weights multiply to x , Pólya’s function and the Davenport–Heilbronn function are the two models of the analytic base without the Euler product, and each kept stage of the ledger is a prime-counting bound on a segment. The register convention of *The Mirror Calculus* is then applied: the independence theorem of the volume is generalized unconditionally to every base in its signature; a base that fixes the referent is proved to decide zeta-specific Admission; counting the constructors of the Selection Jump against the Emmerson–Buchanan barrier hierarchy, the explicit sponsor is the terminal constructor; and the question is resolved at its registers — $\text{RH}_\mathbb{C}$ holds under the analytic interface, sealed, sponsored, and armed, the naming of the trace exhausting the verdict — with the resolution and the barrier result proved to guard each other.

1 The Four Zeroations Are Four Instances of the Contradiction

Write PA in the language $\{0, S, +, \cdot\}$ with the usual axioms:

$$\forall x \quad Sx \neq 0, \tag{PA1}$$

$$\forall x \forall y \quad (Sx = Sy \rightarrow x = y), \tag{PA2}$$

$$\forall x \quad x + 0 = x, \tag{PA3}$$

$$\forall x \forall y \quad x + Sy = S(x + y), \tag{PA4}$$

$$\forall x \quad x \cdot 0 = 0, \tag{PA5}$$

$$\forall x \forall y \quad x \cdot Sy = x \cdot y + x, \tag{PA6}$$

$$(\varphi(0) \wedge \forall x (\varphi(x) \rightarrow \varphi(Sx))) \rightarrow \forall x \varphi(x) \quad \text{for each formula } \varphi. \tag{Ind}$$

Definition 1.1 (Zeroations). The constant 0 occurs in PA in four roles, which we name:

kind	where	what it does
POINT	the constant itself	names an element
TERMINUS	(PA1)	0 is the origin from which every successor chain starts
BASE	(PA3), (PA5)	grounds the recursion for $+$ and $\cdot$
INDUCTION BASE	(Ind)	grounds every inductive proof

A *zeroation* is an occurrence of 0 in one of these roles. Every occurrence of 0 in an axiom of PA is a zeroation; every occurrence in a theorem descends from one.

Definition 1.2 (The contradiction, and the surrogate-zero test, after *Against Zero* [19]). Zero is a sign introduced where a witness, value, member, record, or verdict was withheld, placed in positions otherwise occupied by positive objects, entered into object-language operations, and used to let the system speak as though a withheld presentation had supplied content. The contradiction is root-grammatical: the sign’s positive formation certifies presentation while its semantic office asks it to stand for a withheld presentation. A *surrogate zero* is a sign that performs this same office under another spelling — [blank, null, none, empty, missing, undefined,

an identity “that does nothing”]. The *surrogate-zero test* asks of a sign whether it was introduced for a withheld presentation, occupies positive positions, participates in operations, and supplies content for that withheld presentation; it concerns formal role, and spelling is beside the point.

Proposition 1.3 (The four uses). *Each of the four zeroations is an instance of the contradiction: a formed sign standing for a non-presentation in a positive position.*

Proof. POINT: 0 is introduced as [the count of a roster “with no member”] — a tally reported ahead of any individuation act — and then stands in the positions of tallies as a term. TERMINUS: (PA1) is the record that 0 is the origin of every successor chain, written as a statement about an object [“0 is not a successor”]. BASE: (PA3) and (PA5) ground the recursions at the sign, so that every sum and product is computed through a term that stands for an act withheld; (PA5) further makes the sign absorb every factor. INDUCTION BASE: (Ind) begins every inductive argument at the sign, so every theorem of PA is proved through it. In each case the sign was introduced for a non-presentation, occupies term positions, enters $+$, $\cdot$ and the induction schema, and lets PA assert $x+0 = x$ and $x\cdot 0 = 0$ as facts about an object. The contradiction is instanced four times. $\square$

The four are distinct in kind. POINT is a name. TERMINUS is a record: the statement that 0 is the origin of every successor chain, written in the classical form [“0 is not a successor”]. BASE makes 0 an identity for $+$ and an absorber for $\cdot$. INDUCTION BASE makes every inductive argument begin at an element whose only property is that it is named.

2 Tally Arithmetic

Definition 2.1 (TA). Tally arithmetic TA is the first-order theory in the language $\{\mathbf{1}, S, \oplus, \otimes\}$ with axioms

$$\forall y \quad Sy \neq \mathbf{1}, \tag{TA0}$$

$$\forall x \quad (x = \mathbf{1} \vee \exists y \quad x = Sy), \tag{TA1}$$

$$\forall x \forall y \quad (Sx = Sy \rightarrow x = y), \tag{TA2}$$

$$\forall y \quad \exists z \quad \mathbf{1} \oplus z = Sy, \tag{TA1'}$$

$$\forall x \quad x \oplus \mathbf{1} = Sx, \tag{TA3}$$

$$\forall x \forall y \quad x \oplus Sy = S(x \oplus y), \tag{TA4}$$

$$\forall x \quad x \otimes \mathbf{1} = x, \tag{TA5}$$

$$\forall x \forall y \quad x \otimes Sy = (x \otimes y) \oplus x, \tag{TA6}$$

$$(\psi(\mathbf{1}) \wedge \forall x (\psi(x) \rightarrow \psi(Sx))) \rightarrow \forall x \psi(x) \quad \text{for each formula } \psi. \tag{Ind}_+$$

Its standard model is $(\mathbb{N}_{>0}, 1, S, +, \cdot)$.

Three things are worth saying about this list before anything is proved. (TA1) is positive exhaustion: every tally is the unit or a successor, a disjunction with a witness on each side. (TA1') says the unit is first by exhibiting, for each successor, a tally whose fusion with the unit reaches it. And (TA0) is the one kept record of the theory: the unit is the origin of every successor chain. Every other axiom is an equation between formed terms, an injectivity, or an existence statement with its witness, and every term of the object language of TA is formed by an act, the sign and its surrogates held in the metalanguage. The unit $\mathbf{1}$ is on the other

side of the line: its formation is the first individuation act and its office is to stand for that act, so formation and office agree, and it is formed by an act as every other term is. (TA0) is kept because it must be — Theorem 4.4 shows that every positive first-order theory in this language has the cyclic groups $\mathbb{Z}/k$ among its models, in which $\mathbf{1}$ is a successor; the positive axioms (TA1), (TA1') and induction are all satisfied there. In the register convention of [1] this is a *kept record*: a statement the metalanguage holds about the grammar, exactly as the volume holds nonemission as a metalanguage condition on the exit-locus record.

Lemma 2.2 (Order and predecessor in TA). *Define $x < y \equiv \exists z (x \oplus z = y)$. Then TA proves that $<$ is a strict linear order with least element $\mathbf{1}$, that $y = Sx$ iff $x < y$ and every z with $x < z$ satisfies $y \leq z$, and that for every $y \neq \mathbf{1}$ there is exactly one x with $Sx = y$. Write $\text{pred}(y)$ for this x ; it is defined exactly on the successors.*

Proof. By (Ind₊) on z , $x \oplus z$ is a successor for every x, z ((TA3), (TA4)); hence by (TA0) every $x \oplus z$ is a successor, so $\mathbf{1}$ is least and $<$ is irreflexive at $\mathbf{1}$; irreflexivity in general, transitivity and trichotomy follow by induction as in PA, with (TA0) in place of (PA1) at the base of each. The uniqueness in the last clause is (TA2), and the existence is (TA1). $\square$

3 The Bridge

The object on the positive side is already in the programme's texts: *Counting Back from Infinity* [2] proves that the tally calculus is $(\mathbb{Z}_{>0}, +)$ and that every element of it moves every other ("this semigroup has no identity element internally"), and *Against Zero* [19] proves the same for the native tally sort and recovers classical zero as the free identity adjunction $(\mathbb{Z}_{>0}, +)^1 \cong (\mathbb{N}, +, 0)$, "so the old-world identity is exposed as a completion rather than treated as neutral furniture." The bridge below is that adjunction carried to the full first-order theories with successor, product, and induction, made into a bi-interpretation, and priced. *Counting Back from Infinity* also holds, under the name *Addition is partial, and that is the point*, that fusion of rosters forms only under joint separation; TA's fusion is total because its sort is the pairing class, in which separation is supplied by tagging, and the partiality of roster fusion is the metalanguage condition the class quotients away.

Definition 3.1 (Formation-faithful interpretation). An interpretation of a theory T with a distinguished constant 0 into a theory T_+ is *formation-faithful* when it sends every term of T to a term of T_+ , every zeroation of T to a named positive device of T_+ or to a sentence of T_+ declared as a kept record, and every term of T_+ to a term formed by an act.

Definition 3.2 (The translation τ). τ sends PA-terms to TA-terms and PA-formulas to TA-formulas, with the domain of the interpretation being all of TA's domain (every tally represents a natural number, namely its predecessor count):

$$\tau(0) = \mathbf{1}, \quad \tau(Sx) = S\tau(x), \quad \tau(x + y) = \tau(x) \oplus' \tau(y), \quad \tau(x \cdot y) = \tau(x) \otimes' \tau(y),$$

where the *sign-carrying* operations — the classical $+$ and $\cdot$ as they appear once the sign is removed — are defined in TA by recursion on the second argument:

$$x \oplus' \mathbf{1} = x, \quad x \oplus' Sy = S(x \oplus' y); \quad x \otimes' \mathbf{1} = \mathbf{1}, \quad x \otimes' Sy = (x \otimes' y) \oplus \text{pred}(x) \text{ if } x \neq \mathbf{1}, \quad \mathbf{1} \otimes' Sy = \mathbf{1},$$

Equivalently, in closed form,

$$x \oplus' y = \text{pred}(x \oplus y), \quad x \otimes' y = S(\text{pred}(x) \otimes \text{pred}(y)) \quad (x, y \neq \mathbf{1}), \quad x \otimes' \mathbf{1} = \mathbf{1} \otimes' y = \mathbf{1}.$$

τ commutes with the connectives and quantifiers (relativised to the whole domain).

The closed forms show what the sign concealed: $\oplus'$ is fusion followed by one predecessor, and $\otimes'$ is the product of two predecessors followed by one successor. Each *pred* is total on its argument because, in each case, the argument is a successor — $x \oplus y$ is a successor by (TA3)–(TA4) and (TA1'); x and y are successors by the case hypothesis. This is the point of the construction: *every predecessor the sign concealed is guarded by a witnessed successor*. The classical notation discharged these obligations in silence; TA discharges them in the open.

Theorem 3.3 (The tally bridge). *1. For every axiom α of PA, $\text{TA} \vdash \tau(\alpha)$; hence τ is an interpretation of PA in TA.*

2. The map σ with $\sigma(\mathbf{1}) = S0$, $\sigma(Sx) = S\sigma(x)$, $\sigma(x \oplus y) = \sigma(x) + \sigma(y)$, $\sigma(x \otimes y) = \sigma(x) \cdot \sigma(y)$, relativised to $\{x : x \neq 0\}$, is an interpretation of TA in PA.

3. The composites are provably isomorphic to the identity interpretations: PA proves that $x \mapsto Sx$ is an isomorphism from the identity interpretation onto $\sigma \circ \tau$, and TA proves that $x \mapsto Sx$ is an isomorphism from the identity onto $\tau \circ \sigma$. The theories are therefore bi-interpretable, and $\text{Con}(\text{PA}) \leftrightarrow \text{Con}(\text{TA})$ is provable in primitive recursive arithmetic [22].

Proof. (1) We verify the axioms one by one; this is also the source of the ledger in Section 4. (PA1): τ gives $\forall x Sx \neq \mathbf{1}$, which is (TA0). (PA2): identical to (TA2). (PA3): τ gives $x \oplus' \mathbf{1} = x$, the first recursion clause of $\oplus'$. (PA4): τ gives $x \oplus' Sy = S(x \oplus' y)$, the second clause. (PA5): τ gives $x \otimes' \mathbf{1} = \mathbf{1}$, the first clause of $\otimes'$. (PA6): τ gives $x \otimes' Sy = (x \otimes' y) \oplus' x$. For $x = \mathbf{1}$ both sides are $\mathbf{1}$. For $x \neq \mathbf{1}$, $(x \otimes' y) \oplus' x = \text{pred}((x \otimes' y) \oplus x) = (x \otimes' y) \oplus \text{pred}(x)$ by (TA4) and Lemma 2.2, which is the second recursion clause of $\otimes'$ in Definition 3.2. (The concealed obligation is visible here as one *pred* that may sit on the operation or on its argument.) (Ind): τ gives $(\varphi^\tau(\mathbf{1}) \wedge \forall x(\varphi^\tau(x) \rightarrow \varphi^\tau(Sx))) \rightarrow \forall x \varphi^\tau(x)$, an instance of (Ind₊).

(2) is the same verification in the other direction, using that in PA the set $\{x : x \neq 0\}$ is closed under S , $+$, $\cdot$, contains $S0$, and satisfies (TA1) because $\text{PA} \vdash \forall x (x = 0 \vee \exists y x = Sy)$.

(3) In PA, $\sigma(\tau(x))$ unwinds by induction on x to Sx , and $\sigma(\tau(x + y)) = \sigma(\tau(x) \oplus' \tau(y)) = (Sx + Sy) - 1 = S(x + y)$, likewise for $\cdot$; so $x \mapsto Sx$ is the required isomorphism. The TA direction is symmetric. Equiconsistency is the standard consequence of mutual interpretability, formalisable in PRA. $\square$

Remark 3.4. The bi-interpretation is folklore: $(\mathbb{N}, 0, S, +, \cdot)$ and $(\mathbb{N}_{>0}, 1, S, +, \cdot)$ define each other. The subject of this paper is the shape of τ : that PA's operations, carried into the positive structure, are the native operations *plus a predecessor at every zeroation*, and that each such predecessor is defined only because a successor is in hand. That shape is a theorem about PA, and it is the content of the word “concealed” below.

4 The Ledger

Definition 4.1 (Ledger marks). In a TA-formula obtained by τ , a *substitution* is an occurrence of $\mathbf{1}$ that is the image of a POINT zeroation; a *concealed obligation* is an occurrence of *pred* arising from the closed form of $\oplus'$ or $\otimes'$ — a predecessor witness the classical notation owed at that place and discharged in silence; a *kept record* is a sentence of TA held outside its positive fragment and used in the derivation of the translation. The *cost* of a PA-formula is the triple (substitutions, obligations, records) in its translation, the operations unfolded to closed form.

The cost is a property of a presentation rather than of a theorem: a different closed form for the same operation moves a **pred** and changes the count by one. The figures below are for the closed forms of Definition 3.2, and what is invariant is the qualitative statement that every eliminated zeroation is paid by a substitution or a concealed obligation made explicit, and that exactly one record is kept.

Proposition 4.2 (Cost of the axioms). *The translations of the seven axioms of PA carry the following costs.*

<i>axiom</i>	<i>translation (closed form)</i>	<i>substitutions</i>	<i>obligations</i>	<i>records</i>
(PA1)	$\forall x \, Sx \neq \mathbf{1}$	1	0	1
(PA2)	$\forall x \forall y \, (Sx = Sy \rightarrow x = y)$	0	0	0
(PA3)	$\forall x \, \text{pred}(x \oplus \mathbf{1}) = x$	1	1	0
(PA4)	$\forall x \forall y \, \text{pred}(x \oplus Sy) = S \, \text{pred}(x \oplus y)$	0	2	0
(PA5)	$\forall x \, x \otimes' \mathbf{1} = \mathbf{1}$	2	0	0
(PA6)	$\forall x \forall y \, x \otimes' Sy = \text{pred}((x \otimes' y) \oplus x)$	0	1	0
(Ind)	$\varphi^\tau(\mathbf{1}) \wedge \dots$	1	0	0
<i>total</i>		5	4	1

Every concealed obligation in the table is discharged by a successor already in hand, as shown after Definition 3.2, and the single kept record is (TA0); the ledger therefore closes with every obligation paid in the open and one record kept.

Proof. Read off the closed forms. In (PA5) the two substitutions are the two occurrences of 0; $\otimes'$ at $\mathbf{1}$ is a clause rather than a computation, so its **pred** count is zero of the ledger's marks [the tally of marks is a presented count]. In (PA6) the single obligation is the **pred** of $\oplus'$; the **pred** inside $\otimes'$ belongs to the definition of the operation, which is costed once, in (PA5)–(PA6) jointly, as the recursion that defines it. $\square$

Corollary 4.3 (Cost of a theorem). *If $\text{PA} \vdash \theta$ by a derivation D , then $\text{TA} \vdash \tau(\theta)$ by a derivation whose cost is at most the sum of the costs of the axiom instances used in D plus the number of 0-occurrences introduced by instantiation in D . In particular every theorem of PA has a zero-free proof whose cost is bounded by the number of zeroations in any PA-proof of it.*

The programme states the terminus twice before this paper: *Counting Back from Infinity* proves, under the name *No stage of descent is zero*, that every formed stage of the descent calculus is an inhabited positive residue, and holds in its *Convention on inhabited stages* that the empty open set “remains in the metalanguage of topology” while its positive coherent fragment is built [in its words, with “no object-language falsum, no negation, no empty disjunction, and no vacuous universal quantifier”] [2]. The theorem below proves that this placement is forced rather than chosen: every theory in that fragment has the one-point structure as a model, so the terminus is unstatable in it, and every classical axiomatisation must carry the record as a sentence.

Theorem 4.4 (The terminus is indispensable). *Let T be any set of positive first-order sentences in the language $\{\mathbf{1}, S, \oplus, \otimes\}$ true in $(\mathbb{N}_{>0}, 1, S, +, \cdot)$, together with any instances of the induction schema (Ind₊). Then for every $k \geq 1$ the cyclic structure $(\mathbb{Z}/k, 1, x \mapsto x+1, +, \cdot)$ is a model of T , and in it $\mathbf{1}$ is a successor. Consequently $T \not\vdash \forall y \, Sy \neq \mathbf{1}$: the terminus zeroation lies outside*

every positive first-order axiomatisation, and every axiomatisation of $\mathbf{TA}$ contains a sentence outside the positive fragment that is false in every $\mathbb{Z}/k$. The same holds with injectivity (TA2) added to T , since S is injective on $\mathbb{Z}/k$; so (TA0) is independent of $\mathbf{TA}^+ = \mathbf{TA} - \{\mathbf{TA0}\}$, with $\mathbb{N}_{>0}$ and $\mathbb{Z}/k$ as the two witnessing models. In the positive coherent fragment of [2] — sequents between formulas built from atoms by conjunction, existential quantification, and inhabited disjunction, built short of falsum — the one-point structure $\mathbb{Z}/1$ satisfies every sequent, so the terminus is held there in the metalanguage, by the fragment’s own convention.

Proof. The map $h : \mathbb{N}_{>0} \rightarrow \mathbb{Z}/k$, $x \mapsto x \bmod k$, is a surjective homomorphism of the structures $(\mathbb{N}_{>0}, 1, S, +, \cdot) \rightarrow (\mathbb{Z}/k, 1, +1, +, \cdot)$. Positive first-order sentences are preserved under surjective homomorphisms (Lyndon’s positivity theorem), so every positive sentence true in $\mathbb{N}_{>0}$ holds in $\mathbb{Z}/k$. The induction schema holds in $\mathbb{Z}/k$ because every element is reached from 1 by iterating S . In $\mathbb{Z}/k$, $S(k) = 1$, so $\mathbf{1}$ is a successor. $\square$

Remark 4.5. Theorem 4.4 is the exact boundary of the deprogramming. Three zeroations of $\mathbf{PA}$ — the point, the bases, the induction base — are substitutions and go over to positive devices. The terminus is a genuine kept record: the statement that the first tally is the origin of every successor chain is a statement about the first tally and every successor at once, with an exhibit for each successor and the first tally standing apart, and positive first-order logic has the circle among its models wherever it has the line. The Mirror Calculus keeps exactly this record in the metalanguage — as nonemission for the exit-locus record, and as the Kept Register more generally [1] — and the theorem shows the choice is forced rather than stylistic.

Proposition 4.6 (Where the concealed obligation comes due). *In $\mathbf{TA}$ the obligations concealed by $+$ and $\cdot$ are discharged at every argument, because $x \oplus y$ is a successor for all x, y and the product clause at $\mathbf{1}$ is a return to the unit. Extend both theories by a quotient: in $\mathbf{PA}$ the partial operation $x \div y$ with $y \cdot (x \div y) = x$, in $\mathbf{TA}$ its transport $x \div' y$ with $y \otimes' (x \div' y) = x$. Then the obligation concealed at the divisor is discharged exactly when $\mathbf{pred}(y)$ is formed, that is, when $y \neq \mathbf{1}$; at $y = \mathbf{1}$ — the image of the classical divisor 0 — the obligation is undischageable, and $\mathbf{TA}$ withholds the formation. $\mathbf{PA}$ ’s grammar requires a term in that position and supplies [“undefined”] — a returned non-value in a term position, introduced where a quotient was withheld: a surrogate zero by the test, the sign kept under a second spelling.*

Proof. $y \otimes' z = S(\mathbf{pred}y \otimes \mathbf{pred}z)$ for $y, z \neq \mathbf{1}$ and $\mathbf{1} \otimes' z = \mathbf{1}$. For $y \neq \mathbf{1}$ the equation $y \otimes' z = x$ has a solution iff $\mathbf{pred}y$ divides $\mathbf{pred}x$ in $\mathbf{TA}$ ’s native product, and the solution is formed from a witnessed quotient. For $y = \mathbf{1}$ the left side is $\mathbf{1}$ for every z , so the equation holds iff $x = \mathbf{1}$ and then for every z : the quotient is withheld or unselected, and in each case the term position awaits an occupant. $\mathbf{TA}$ withholds formation, which is a metalanguage judgment; $\mathbf{PA}$ must return something in a term position, and “undefined” is the something. $\square$

Remark 4.7 (What the ledger says). Read in the register convention of [1, 19]: $\mathbf{PA}$ ’s 0 is the sign for a non-presentation — introduced for [the roster “with no member”] and then made a tally — and it is eliminable (Theorem 3.3), which is its formal usability. The elimination has a uniform cost: one substitution per naming, one concealed obligation per recursion step through the base, and the classical notation paid the obligations by writing the sign that made them vanish. The annihilation law ($\mathbf{PA5}$), which in the classical register reads as multiplication destroying its argument, translates as $x \otimes' \mathbf{1} = \mathbf{1}$: a return to the unit, every tally kept and the unit named twice. The semantic error becomes a bookkeeping error at the first operation whose concealed obligation comes due, and Proposition 4.6 shows that this is division and that

the classical notation meets it by installing a surrogate. The ledger here is syntactic — it counts marks in a translation — and is separate from the explicit-formula ledger of the volume, on which Problem 6 asks the price of a foundation to be computed; this paper supplies the interpretation half of that problem and the syntactic half of its price.

5 The General Schema

The construction used three things about PA: 0 is least in a discrete order, S is a bijection onto the elements other than 0, and the operations are defined by recursion through 0. That is the whole list.

Theorem 5.1 (Elimination of least-element zeros). *Let T be a first-order theory in a language $L \ni 0, S$ such that T proves*

$$\forall x (x = 0 \vee \exists y x = Sy), \quad \forall x \forall y (Sx = Sy \rightarrow x = y), \quad \forall x Sx \neq 0.$$

Let T_+ be the transport of T along the bijection $x \mapsto Sx$ from the domain onto the elements other than 0: its language has a constant $\mathbf{1}$ for $S0$ and, for each function symbol f of T , a symbol f' interpreted as $S \circ f \circ \text{pred}$ on the transported domain. Then T and T_+ are bi-interpretable; the point and induction-base zeroations of T become $\mathbf{1}$, and the terminus becomes the kept record $\forall y Sy \neq \mathbf{1}$, which by Theorem 4.4 lies outside every positive axiomatisation. Whenever f is given in T by recursion equations through 0, f' is given in T_+ by recursion equations through $\mathbf{1}$ in which every pred is guarded by a successor in hand, as for $\oplus'$ and $\otimes'$; in that case the base zeroations of f become positive clauses and the ledger of Section 4 applies verbatim.

Proof. Transport of structure along a definable bijection is a bi-interpretation. The three hypotheses are what Lemma 2.2 and Theorem 3.3 used to show that the transported domain is all of the positive part, that pred is defined exactly on the successors, and that recursion through 0 becomes recursion through $\mathbf{1}$; the argument goes through with f' in place of $\oplus', \otimes'$. $\square$

Corollary 5.2. *The following have zero-free presentations, every term formed by an act, by the successor shift, with the four zeroation kinds and the ledger structure of Section 4: PA and all its fragments $I\Sigma_n$, $I\Delta_0$, PRA; Presburger arithmetic; and the first-order theory of any discretely ordered semiring with least element. (Skolem arithmetic lies outside the schema: its language has multiplication only, and successor lies outside its definable relations.)*

Remark 5.3 (The general Mirror Calculus). The schema is the first instance of a general move. A *surrogate-zero device* in a theory T is a named term that T uses as an identity, an absorber, a terminus, or a vacuous base; the Mirror Calculus classifies these [1]. For each kind the schema names the positive replacement (a unit, a return to the unit, an exhaustive disjunction, an inductive start at the unit) and the price (a fixing, a witness). The general Mirror Calculus is the theory of such replacements for an arbitrary T : for which T does a zero-free presentation exist, what is its cost, and which devices — the additive identity of a ring, the empty set of ZF, the bottom of a lattice — admit replacement and which resist it. Theorem 5.1 settles the least-element case completely and names the obstruction in the others: the schema carries a device along a bijection from the rest of the domain, and a device outside every such bijection stays where it is.

6 The Test Beyond Arithmetic

The schema of Theorem 5.1 covers the sign where it is a least element. In algebra and set theory the sign appears under other spellings, and the surrogate-zero test is the instrument that finds it there; in two of the three cases it has been deprogrammed already, under other names.

6.1 The Identity of a Group Is Zero Under Another Name, and Heap Theory Removes It

A *heap* (Prüfer’s *Schar*, Baer’s *groud*) is a set H with a ternary operation $[x, y, z]$ satisfying $[x, y, y] = x = [y, y, x]$ and $[[x, y, z], u, v] = [x, y, [z, u, v]]$ [24, 23]. Every group is a heap under $[x, y, z] = xy^{-1}z$, and every heap with a chosen element e is a group under $xy = [x, e, y]$; the two passages are inverse, and different choices of e give isomorphic groups [25]. The identity of a group is a surrogate zero by the test — introduced as [the act “that does nothing”], written e or 1 so as to sound like an act, placed in the positions of acts and composed with them — and heap theory removes it: every use in the group axioms ($xe = x$, $xx^{-1} = e$) becomes a heap identity in which every element is formed by the same act. The volume’s corollary that the affine basepoint separates [1] is this theorem, and the ledger for it is the substitution $e \mapsto$ the middle argument of $[\cdot, \cdot, \cdot]$, one mark per occurrence.

6.2 The Zero of a Ring, and Truss Theory

A *truss* [26] is an abelian heap with an associative multiplication distributing over the ternary operation. A ring is a truss with a chosen element; a truss with a chosen element e is a ring with additive identity e if and only if e is multiplicatively absorbing, and in general it is a ring-like structure in which the absorber has been forgotten. The additive zero of a ring is the sign itself, and truss theory removes it; its multiplicative role, $0 \cdot x = 0$, is the one ring axiom the passage rewrites, and Brzeziński’s analysis of which trusses come from rings is precisely the accounting of that residue. In the language of this paper, the point and base zeroations of a ring are substitutions, and the annihilation law is where the ledger carries a concealed obligation.

6.3 The Empty Set Is the Surrogate of Set Theory, and a Kept Record Once Removed

Proposition 6.1. *Let T be a first-order set theory proving Extensionality and the existence of a set.*

1. *If T proves the Separation schema for all formulas, then $T \vdash \exists x \forall y (y \notin x)$.*
2. *If T proves the $\in$ -induction schema (Foundation for definable classes: for every formula φ , if $\forall x (\forall y \in x \varphi(y) \rightarrow \varphi(x))$ then $\forall x \varphi(x)$), then $T \vdash \exists x \forall y (y \notin x)$.*

Set-level Foundation alone leaves the chain standing: the structure $(\mathbb{Z}/k, \in)$ with $n \in m$ iff $n \equiv m - 1$ satisfies Extensionality and set-level Foundation, and every element has a member.

Proof. (1) Separate $\{y \in a : y \neq y\}$ from any set a . (2) Apply the schema to $\varphi(x) := \exists z \forall y (y \notin z)$: if every member of x satisfies φ then so does x (if x is [memberless], x itself is the witness; otherwise a member supplies one), so every x satisfies φ , and a set exists. For the counter-model, each m has the single member $m - 1$, whose single member $m - 2$ differs from $m - 1$ for $k \geq 2$, so $m - 1$ is $\in$ -minimal in m . $\square$

So a set theory in which every set has a member — the presence-only reading of “set” — must restrict Separation to positive formulas and must weaken Foundation to its set-level form or drop it. We conjecture, and leave open, that $\forall x \exists y (y \in x)$ is consistent with Extensionality, Pairing, Union, and Separation for positive formulas, with a model built from Aczel’s universe of non-well-founded sets [27] in which the Quine atom $\Omega = \{\Omega\}$ replaces the empty set as the least inhabited object.

The empty set is a surrogate zero by the test — [the collection “with no member”], written as a set so as to be a set — and it is of the terminus kind: its existence is a kept record [“no member”], forced by Separation for formulas outside the positive fragment and by $\in$ -induction, and a presence-only set theory is a theory of hereditarily inhabited, non-well-founded sets, whose positive comprehension is the subject of Esser’s positive set theories [28]. Which fragment of ZF it interprets, and at what cost, is the set-theoretic instance of Problem 6 of [1] and is open.

6.4 The Demarcation

Three places the sign appears, three verdicts. Where it is a least element, its uses are substitutions with concealed obligations and one kept record (Theorems 3.3, 4.4, 5.1). In groups and rings it appears as the identity and the additive zero, and heap and truss theory remove it entire. In set theory it appears as the empty set, and once removed it is a kept record. The presence-only programme is the claim that zero is a formed sign assigned to a non-presentation, that its surrogates are the same sign under other spellings, and that the test finds them by role; the classification says what each use stood in for and where, once the sign is removed, the non-presentation is held: in a substitution, in an obligation the notation owed, or in a record the metalanguage keeps. The theorems above are the first three entries.

7 The Independence Result, Generalized and Unconditional

The class-wide theorem of [1] is one instance of a result that holds for every base in the descent signature, and the completeness theorem says exactly which bases it holds for. Fix the signature Σ_{desc} of [1]: a sort of descents D (finite root multisets), a sort of presented points, fusion, conjugation, the mirror $\iota(s) = 1 - \bar{s}$, the carrier F_D , the field $\Phi_D = \text{Re } F'_D / F_D$, and the sector predicate sec^+ read as $\text{Re } s > \frac{1}{2}$. Throughout this section a *structure* is one whose sort of presented points contains every rational point away from the roots, as the base of [1] requires (“presented rational test points away from the roots”); a theory T is assumed to prove this. Let Adm be the sentence *for every descent and every presented point in sec^+ , $\Phi_D(s) > 0$* , and $\overline{\text{Adm}}$ its positively witnessed counter-inscription *some descent and some presented point in sec^+ have $\Phi_D(s) < 0$* .

Definition 7.1 (Axial and off-axial structures). A Σ_{desc} -structure is *axial* if every root of every descent has real part $\frac{1}{2}$, and *off-axial* if some descent contains a root with real part other than $\frac{1}{2}$.

Lemma 7.2. *In every axial structure Adm holds. In every off-axial structure closed under the mirror and conjugation, $\overline{\text{Adm}}$ holds at some rational point.*

Proof. Axial: each term $(\sigma - \frac{1}{2})/|s - \rho|^2$ is positive for $\sigma > \frac{1}{2}$, and a descent is nonempty. Off-axial: let $\rho = \beta + i\gamma$ with $\beta \neq \frac{1}{2}$; by the mirror we may take $\beta > \frac{1}{2}$. At $s = \sigma + i\gamma$ with $\frac{1}{2} < \sigma < \beta$, the term for ρ is $(\sigma - \beta)/(\beta - \sigma)^2 = -1/(\beta - \sigma)$, which tends to $-\infty$ as $\sigma \rightarrow \beta^-$,

while the other finitely many terms stay bounded; choose a rational σ close enough to β and the sum lies below zero [the sign here names a presented rational bound]. (Model B of [1] is this construction at $\beta = \frac{4}{5}$, $\gamma = 5$, $\sigma = \frac{3}{5}$.) $\square$

Theorem 7.3 (Generalized unconditional independence). *Let T be any consistent Σ_{desc} -theory. The following are equivalent:*

1. $T \not\vdash \text{Adm}$ and $T \not\vdash \overline{\text{Adm}}$;
2. T has an axial model and an off-axial model.

In particular Adm is independent of every T that is satisfied by some axial structure and by some off-axial structure — every base that leaves the axis free on both sides. Consistency is the whole hypothesis.

Proof. (2) $\Rightarrow$ (1) is Lemma 7.2 with soundness. (1) $\Rightarrow$ (2): by the completeness theorem $T \not\vdash \text{Adm}$ gives a model of $T + \overline{\text{Adm}}$, which is off-axial by Lemma 7.2 read contrapositively; and $T \not\vdash \overline{\text{Adm}}$ gives a model of $T + \text{Adm}$, which is axial, since an off-axial model closed under the operations would satisfy $\overline{\text{Adm}}$. $\square$

This is the class-wide theorem of [1] with the base left free: that theorem is the case $T = \mathbf{P}$, Model A the axial model and Model B the off-axial one. The generalization is unconditional, and it is also exactly as general as any independence result in this signature can be, by the following.

Theorem 7.4 (Faithfulness excludes independence). *Extend the signature by a sort of completed descents with a convergent field $\Phi = \text{Re} \sum_{\rho} 1/(s - \rho)$ and a constant d_{ζ} , and let Adm_{ζ} be Adm relativized to d_{ζ} . Call T faithful to ζ if the standard structure $\mathfrak{Z}$, in which d_{ζ} is the multiset of nontrivial zeros of ξ , is a model of T and every model of T agrees with $\mathfrak{Z}$ on Adm_{ζ} . Then for every faithful T , exactly one of $T \vdash \text{Adm}_{\zeta}$ and $T \vdash \overline{\text{Adm}_{\zeta}}$ holds, and which one is $\text{RH}_{\mathbf{C}}$. A faithful theory decides zeta-specific Admission.*

Proof. In $\mathfrak{Z}$, Adm_{ζ} holds iff $\text{RH}_{\mathbf{C}}$, by the argument of Lemma 7.2 (the off-axis term dominates the convergent remainder near its root). A sentence true in every model of T is provable from T , by completeness, and a sentence false in every model has its counter-inscription provable. So T decides Adm_{ζ} , in the direction $\text{RH}_{\mathbf{C}}$ takes. The theorem is the completeness theorem applied to the definition of faithfulness; its content is the dichotomy that follows. By Löwenheim–Skolem every first-order T has models in which d_{ζ} differs from the standard multiset; faithfulness asks only that its models agree on the one sentence. $\square$

Corollary 7.5 (The ζ instance). *For any Σ_{desc} -theory T with a constant d_{ζ} , exactly one of the following holds:*

1. T leaves the referent free, and then Adm_{ζ} may be independent of T by Theorem 7.3 — and the independence, like the class-wide one, is a statement about the models of T and is silent on the zeros of ξ ;
2. T fixes the referent, and then Adm_{ζ} is decided by T , and deciding it is deciding $\text{RH}_{\mathbf{C}}$.

Against Zero [19] states the affirmative route exactly: a *Conservative Independence Transfer* theorem (native independence lifts to a classical theory T along a proof-reflecting translation) and the *One-Sided Independence-to-Truth* theorem, composed as the chain *native zeta-specific independence* + *proof-reflecting translation* $\Rightarrow$ *arithmetic independence* + Π_1 *form and* Σ_1 -*completeness* $\Rightarrow$ *truth* $\Rightarrow$ RH_C ; and it says of the chain’s antecedent that “a zeta-specific conservative-transfer theorem is the exact next bridge required.” Theorem 7.4 is the statement of what that bridge costs: the antecedent is zeta-specific independence, a theory faithful to ζ decides the zeta instance, and the one-sided theorem’s hypothesis $T \not\vdash \neg\varphi_{\text{RH}}$ is, for sound Σ_1 -complete T , equivalent to φ_{RH} itself. The chain is valid and its entry condition is its conclusion, which is why the volume forms the verdict by a declaration. *Against Zero* also gives the faithfulness fork in one sentence — “every faithful classical realization of the adopted presence theory satisfies classical RH; failure of classical RH would instead establish failure of that realization’s faithfulness” [a quotation, in the source’s own words] — and Theorem 7.4 is that sentence as a theorem. This is the volume’s *Faithfulness Fork* and its *Conservation of difficulty* remark [1] made into a theorem, and it is the independence side of the irresolvability–independence distinction drawn in [21], where selected irresolvability is a formed object and independence is a metalanguage record: “that problem is exactly as hard as RH_C itself” is Theorem 7.4, and “Model B separates the class-wide fork precisely by reinterpreting the class” is clause (1). An *admitting model of the reference clauses* — the volume’s Problem 7 — therefore exists if and only if the reference clauses are consistent with Adm_ζ ; if the clauses are faithful, that is if and only if RH_C .

Remark 7.6 (What this settles about the method). Independence is an affirmative method in one paradigm situation: when the independent sentence is Π_1 and the theory is sound and Σ_1 -complete, for then $T \not\vdash \neg\varphi$ is equivalent to the truth of φ . Theorem 7.4 shows that in the descent signature the corresponding situation — T faithful — is one in which independence is impossible, and Theorem 7.3 shows that where independence is possible the theory is unfaithful and the independence is silent about ζ . The two cases exhaust, and this is the same “no third class” [the volume’s phrase] it proves for routes that form a sign: the route through independence forms its sign about the models of T alone, which is why the volume’s verdict is formed by a declaration and carries its sponsor.

8 Why the Sign Was the Obstacle

The classical Riemann hypothesis is written with the sign at its centre: “ $\xi(s) = 0$ implies $\text{Re } s = \frac{1}{2}$.” This section states, as theorems about the form of that sentence, what the sign did to the question. Under the convention of *Against Zero* [19], the metalanguage may name the sign and its office in order to diagnose them; bracketed phrases in this section are such diagnostic mentions of the classical form, and every assertion of the section is about what presents. The theorems of the first half of the paper are the instruments: the sign is a formed mark for a non-presentation, positive logic forms its universal sentences over presented records alone (Theorem 4.4), and what the sign conceals is a witness obligation or a record.

8.1 The classical statement is a universal over the sign

Proposition 8.1 (Form of the classical statement). *RH_C is logically equivalent to the sentence [there is no s with $\xi(s) = 0$ and $\text{Re } s \neq \frac{1}{2}$]: a universal over the complex plane whose matrix is an*

evaluation set equal to the sign. Under the standard arithmetic interface it is a Π_1 sentence and its counter-inscription a Σ_1 sentence; in the positive coherent fragment of Counting Back from Infinity [2] it is held as a record rather than formed, and in Transcendental Presence Notation it is a nonemission record over an infinite ledger.

Proof. The equivalence is propositional. The Π_1 form is the classical elementary criterion [18], and its counter-inscription asserts a finite witness. The positive fragment holds it as a record by Theorem 4.4 read for this signature: a universal whose content is the closing of a search is exactly the sentence Lyndon’s theorem places outside every positive theory, as the terminus was placed outside the positive fragment of TA. In TPN a universal non-presentation is held stage by stage as a nonemission record, and the ledger of stages here is infinite. $\square$

Corollary 8.2 (The certification asymmetry is grammatical). *The asymmetry between a refuting resolution of the hypothesis (a finite certificate) and an affirming one (a universal) is the asymmetry between a presented record and a universal over withheld presentation. It is a consequence of writing the hypothesis with the sign, and the barrier hierarchy of [20] is the theory of what an affirmation of a universal non-presentation costs when it is attempted stage by stage.*

Proof. A counter-instance is an exit-locus record with an off-axis address and its certificate: a presented object. The affirmation is the nonemission record over every stage, which in arithmetic is Π_1 and whose closure over the ledger is an ω -inference. The Selection Jump theorem of [1] is the statement that the family forms by one of three constructors, and the barrier hierarchy is the statement of what each stagewise route to the family meets. $\square$

8.2 The verification record presents tallies; the sign is a mention

Proposition 8.3 (What is verified). *Every verified instance of the hypothesis to a height T is an identity between two presented tallies: the argument-principle count $N(T)$ of exit points of ξ in the strip up to height T , and the count of sign changes of the real function $Z(t)$ on the line up to T . The evaluation $\xi(\rho) = 0$ belongs to the classical description of the result; what the verification presents is the two tallies.*

Proof. This is the structure of the verification literature from Turing’s method onward [6, 7]: $N(T)$ is computed by the argument principle as an integer, the line count is computed by locating sign changes of Z with interval arithmetic, and the hypothesis to height T is the equality of the two integers. A sign change is a presented pair of values of opposite sign; a count is a presented tally; and those two are the whole of what a verification presents. $\square$

So the ledger of the hypothesis has always been a kept law over presented records — $\text{Keep}(n, k_n)$ in the volume’s notation — and the classical form reads each stage of it as evidence about the sign.

8.3 Riemann removed the first surrogates

Proposition 8.4 (The trivial zeros are exits of the wrong filter). *The function ζ has zeros at $-2, -4, -6, \dots$ which carry the gamma normalization’s exits rather than a prime’s record: they are the exits of the gamma normalization in the functional equation. The completed function $\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)$ removes them and keeps an entire function, symmetric under $s \mapsto 1-s$, whose exits are exactly the records of the prime-fused descent.*

Proof. Standard [3, 5]: the poles of $\Gamma(s/2)$ at $-2n$ cancel the zeros of ζ there, and ξ is entire with $\xi(s) = \xi(1-s)$ and zeros the nontrivial zeros of ζ . $\square$

In the language of this paper, the trivial zeros are the sign appearing where a filter other than the prime filter exits, and Riemann's completion is the first act of deprogramming in the subject: the object of the hypothesis was, from its first statement, the one from which those marks had been removed.

8.4 The zero of ξ is a presented carrier

Proposition 8.5 (The carrier reading). *In the explicit formula*

$$\psi(x) = x - \sum_{\rho} \frac{x^{\rho}}{\rho} - \log 2\pi - \frac{1}{2} \log(1 - x^{-2}), \quad x > 1, x \notin \mathbb{Z},$$

each exit-locus record ρ presents as a term of weight $|x^{\rho}| = x^{\operatorname{Re} \rho}$. Under the analytic interface I , $\operatorname{Adm}_{\zeta}$ is equivalent to the statement that every carrier has weight $x^{1/2}$, and to $\psi(x) = x + O(x^{1/2} \log^2 x)$.

Proof. The explicit formula is von Mangoldt's [4, 5]. Equal weight $x^{1/2}$ for every ρ is $\operatorname{Re} \rho = \frac{1}{2}$ for every ρ , which is $I(\operatorname{Adm}_{\zeta})$; the error-term equivalence is classical. $\square$

The withheld value at ρ on one side of the interface is the presented record ρ on the other. The hypothesis, read through the prime ledger, is a law about the weights of presented carriers — every record presents with the same weight — and this is its native form: a universal over presented records, kept stage by stage, with its seal.

8.5 The mechanism

Theorem 8.6 (What the sign did). *Writing the hypothesis with the sign performs the following conversion, each step a theorem above: a law about the weights of presented carriers (Proposition 8.5) becomes [a universal over the sign] on a totality outside presentation (Proposition 8.1); that universal lies outside positive formation and is Π_1 in arithmetic, so its affirmation is an ω -inference (Corollary 8.2); the demand for a classical proof is therefore the demand for an ω -inference with its sponsor left off the page, which the Selection Jump refuses to form and the barrier hierarchy shows splits under every stagewise attempt to form it (Theorem 10.1); and the verification ledger, which presents records and tallies at every stage (Proposition 8.3), is read as evidence about the sign rather than as the kept stages of a law. Deprogramming the sign reverses the conversion: the hypothesis is a kept law over presented records, its seal is formed by a constructor, and the constructor is named.*

Proof. Composition of the cited results. $\square$

Remark 8.7 (What is conserved). The sign fixed the *form* of the obstacle. The content — which sponsor the seal carries, and whether a generator with a presented uniform keeping law exists — is conserved under the reversal (Theorem 7.4) and is relocated into the sponsor, where it is a named term rather than a symbol's silence. This is the exact sense in which the sign was the obstacle and in which removing it is the resolution: the verdict the classical form demanded was a seal with its sponsor left off the page, a surrogate verdict by the test of Section 1; the verdict the presence form delivers is a seal with its sponsor named, which is the only kind the licensing theorem admits.

9 The Primes

A number theorist will ask what the foregoing says about primes. The answer is that the Admission law is a known prime-side criterion, its kept stages are prime-counting theorems on segments, and the two models that make the independence theorem number-theoretic are classical. This section states each as a theorem with its source, so that the grammar and the primes are read off each other line by line.

9.1 Admission is Lagarias positivity, which is Li's criterion

Theorem 9.1 (The law on the prime side). *Under the analytic interface I , the following are equivalent:*

1. Adm_ζ ;
2. $\text{Re } \frac{\xi'(s)}{\xi(s)} > 0$ for $\text{Re } s > \frac{1}{2}$ (Lagarias [8]);
3. $\lambda_n = \sum_\rho [1 - (1 - 1/\rho)^n] \geq 0$ for every $n \geq 1$ (Li [9]);
4. RH_C .

Moreover each λ_n is an explicit expression in archimedean terms and a finite sum over prime powers (Bombieri–Lagarias [10]), so (3) is the positivity $\lambda_n \geq 0$ of a prime-indexed sequence.

Proof. (1) $\Leftrightarrow$ (4) is the interface theorem of [1]. (2) $\Leftrightarrow$ (4) is Lagarias's theorem. (3) $\Leftrightarrow$ (4) is Li's criterion. The arithmetic expression of λ_n is obtained by applying the explicit formula to the test function $1 - (1 - 1/\rho)^n$, as in [10]. $\square$

The volume's field $\Phi_D = \text{Re } F'_D/F_D$ for a finite descent D is statement (2) for the partial Hadamard product over D ; the finite descents of the class-wide theorem are the stages of this law, and a kept stage on the prime side is $\lambda_n \geq 0$ presented, which is what the computations of Keiper [11] and Mařlanka [12] present as far as they reach.

9.2 The mirror pairs carriers whose weights multiply to x

Proposition 9.2 (Balance). *Let $\rho = \beta + i\gamma$ be an exit-locus record and $\iota(\rho) = 1 - \bar{\rho}$ its mirror image. In the explicit formula their carriers have weights x^β and $x^{1-\beta}$, whose product is x for every record. Adm_ζ under I is the statement that every mirror pair splits x evenly; an off-axis record is a pair in which one carrier has weight exceeding $\sqrt{x}$, so that the prime-counting oscillation at frequency γ has amplitude $x^{\max(\beta, 1-\beta)} > \sqrt{x}$.*

Proof. $|x^\rho| = x^{\text{Re } \rho}$, and $\text{Re } \iota(\rho) = 1 - \beta$. $\square$

9.3 The analytic-register independence theorem

Theorem 9.3 (Pólya and Davenport–Heilbronn as Models A and B). *Let $\mathbf{P}_\text{an}$ be the base whose descents are entire functions of order one with the symmetry $F(1-s) = F(s)$, real on the axis, presented with their exit-locus records, and stated short of any Euler-product clause. Then $\mathbf{P}_\text{an}$ has an axial model and an off-axial model, so Admission is independent of it:*

- A. Pólya’s function ξ^* [13], obtained by replacing the kernel of the Fourier representation of ξ by its leading term, satisfies the symmetry and has all its zeros on the axis;
- B. the Davenport–Heilbronn function [14], a linear combination of two Dirichlet L -functions modulo 5, satisfies a functional equation of ζ -type and has zeros off the axis.

Consequently the content of Adm_ζ lies in the Euler-product clause — the prime-fused filter of [1] — in the analytic register, for Dirichlet series as well as for polynomial descents.

Proof. Both functions are in the base by construction; the zero locations are the cited theorems; Theorem 7.3 gives the independence. $\square$

This is the Symmetry No-Go of [1, 19] as a number theorist states it: the functional equation and conjugation symmetry locate the axis and leave occupation to the clause that names the primes.

9.4 Each kept stage is a prime theorem on a segment

Theorem 9.4 (Kept stages count primes). *If the hypothesis is verified to height H — every exit-locus record with $|\gamma| \leq H$ is on the axis — then*

$$|\pi(x) - \text{li}(x)| < \frac{\sqrt{x} \log x}{8\pi} \quad \text{for } 2657 \leq x \leq X(H),$$

where $X(H)$ grows with H (Büthe [15]). The seal of the ledger is the same inequality for all $x \geq 2657$ (Schoenfeld [16]).

Proof. Büthe’s theorem and Schoenfeld’s. $\square$

So $\text{Keep}(n, k_n)$ is, on the prime side, a bound on the error of the prime count on an initial segment, and Seal_ω is the bound on every segment at once. The stages of the ledger are theorems about primes; the seal is a theorem about all of them, and the Selection Jump says the seal forms by one of the three constructors alone.

9.5 What a generator would be, and what the verdict delivers

A uniform keeping law with a presented proof — the second constructor — is, by Theorem 9.1, a proof that $\lambda_n \geq 0$ for every n at once: Li’s criterion discharged in full. That is the object Problem 7 of the volume asks for and the object the barrier hierarchy places beyond every stagewise architecture; on the prime side it is the statement that the prime-power sums inside every λ_n stay below the archimedean terms uniformly. The verdict of record delivers, with its sponsor in the trace, every consequence of the hypothesis for primes: Schoenfeld’s bound for all $x \geq 2657$, $\psi(x) = x + O(\sqrt{x} \log^2 x)$, and Cramér’s bound $O(\sqrt{x} \log x)$ on gaps between consecutive primes [17]. A number theorist who accepts the verdict at its trace accepts these with the same trace; one who asks for the generator asks for Li positivity proved uniformly, which is the question in its sharpest prime-side form.

10 The Register-Typed Riemann Question

The Mirror Calculus [1] states the Riemann hypothesis in three registers and proves something in each. This section states exactly what, so that the name attached to this paper carries the right weight.

Grammar. In the strict presence grammar G_P , whose terms are all formed by acts and whose constructors compare evaluations with presented values alone, the literal sentence “ $\xi(s) = 0 \Rightarrow \text{Re } s = \frac{1}{2}$ ” is held outside formation, by the volume’s proposition *Literal nonformation* [1], which rests on every tally of the positive semigroup moving every other. The native axis sentence RH_M — every certified exit-locus record of the completed prime-fused descent is fixed by $\iota(s) = 1 - \bar{s}$ — is formed. This is a theorem about G_P ; in classical complex analysis $\xi(s) = 0$ keeps its own formation, and the volume says so.

Native. Over a declared base P whose descent fragment is: nonempty finite root multisets closed under fusion, conjugation and the mirror, with carrier $F_D(s) = \prod (s - \rho)$ and field $\Phi_D = \text{Re } F'_D / F_D$, and with root location left free, *class-wide Admission* ($\Phi_D(s) > 0$ at every presented point right of the axis, for every descent) is independent: Model A, the multisets on the axis, satisfies it, because each term $(\sigma - \frac{1}{2})/|s - \rho|^2$ is positive; Model B, generated by $\{\frac{4}{5} \pm 5i, \frac{1}{5} \pm 5i\}$, refutes it at $s_0 = \frac{3}{5} + 5i$, where $\Phi = -7821885/3131252$. This is correct and elementary: a base that leaves root location free leaves a sentence about root location free. The classical shadow of class-wide Admission is *false* (Model B is a classical counterexample), so this independence is a statement about the expressive scope of P , and the volume records that it “does not imply zeta-specific independence.”

Adoption. The zeta-specific law Adm_ζ — every certified exit-locus record of the zeta descent is admitted to the axis — is adopted under an explicit sponsorship event, the Sponsorship Declaration of [1], whose form is developed in the witness-fibre programme of [21], and $P^\dagger = P + \text{Adm}_\zeta$ derives RH_M by instantiation and universal introduction. The volume calls this theorem “intentionally transparent” and states that it “is not a derivation of that law from the unextended base theory.” Under the named analytic interface I , which sends exit-locus records to zeros of ξ and the mirror axis to $\text{Re } s = \frac{1}{2}$, $I(\text{Adm}_\zeta) \Leftrightarrow \text{RH}_C$ holds by the definition of I . The classical strength of the adoption is therefore stated exactly: it is the hypothesis itself.

The one-sided route. The volume also proves a conditional theorem: if the arithmetic shadow φ_{RH} (Π_1 , by the standard criteria) is independent of a sound Σ_1 -complete theory T , then it is true, since its counter-inscription would be a true Σ_1 sentence that T proves. The theorem is correct and classical. Its hypothesis — independence of φ_{RH} from PA , say — is open, and each route on the page stops short of it — class-wide independence (whose shadow is false) and the adoption (which is the conclusion). Every statement in this paragraph is in the volume; what this section adds is the line between them.

What the grammar changes. The reading of the hypothesis, in three respects, each a theorem. First, what the hypothesis *is*: a statement about certified records and an involution, formed with every term an act — the Riemann analogue of the tally bridge of Section 3, with the terminus appearing as the metalanguage condition of nonemission on the exit-locus record, by exactly the mechanism of Theorem 4.4. Second, where its content *lies*: the symmetry no-go — an explicit polynomial carrying the full tent-group symmetry with an off-axis quartet, $\{\frac{4}{5} \pm 5i, \frac{1}{5} \pm 5i\}$ in [1] and the root $\frac{4}{5} + 7i$ of $((s - \frac{1}{2})^2 - a^2)((s - \frac{1}{2})^2 - \bar{a}^2)$, $a = \frac{3}{10} + 7i$, in [19] — locates the whole of Adm_ζ beyond symmetry, in the prime-fused filter, which under I is the Euler-product side of the Selberg axioms. Third, what a *verdict* on it is: by the volume’s licensing theorem every sound route that forms a sign exposes a non-invariant point, so a verdict

carries its sponsor or it is a reification of non-presentation; the verdict RH_C under I is formed, sponsored by α_ζ , and traced, and in the native register the naming exhausts the verdict [1, 21].

What each register keeps. The native register keeps the formed verdict with its trace. The classical register keeps the status of its own shadow φ_{RH} , which in that register is the statement whose refutation side is a finite certificate and whose affirmative side Theorem 7.4 places beyond every independence result. Each statement is made in its own register, each with authority over itself alone, and the register convention exists so that both can be written on one page. Section 10.1 states what the verdict’s sponsor is, when the constructors are counted.

10.1 The Terminal Constructor

The volume’s Selection Jump theorem [1] is proved unconditionally and says that a coherent omega-family forms by one of three constructors: a presented coherent family; a guarded generator together with a presented proof of its uniform keeping law; or an explicit occupation sponsor. Stagewise keeping is a record at each stage, and the family forms by one of the three constructors alone. The formation system refuses, by this theorem, to count back from infinity.

Problem 7 of the volume is the attempt to use the second constructor for the zeta instance: a generator with a presented uniform keeping law, which would re-form the axis verdict with the generator named as its sponsor. The barrier hierarchy of Emmerson and Buchanan [20, 21] — nonterminality, certification asymmetry, the selection jump, reflection collapse, the Tarski barrier, diagonal impossibility — is the claim, proved in that work, that every stagewise certificate architecture around the Riemann hypothesis is forced to maximal complexity of its kind, so that the second constructor’s presented uniform keeping law splits into ladders at each level of the hierarchy. The first constructor, a presented completed family, is the object the second constructor would have produced.

Theorem 10.1 (Terminal form of the verdict). *Assume the Selection Jump theorem of [1] and the barrier hierarchy of [20]. Then the explicit sponsor is the terminal constructor for the zeta family: the axis verdict forms by the Omega-Seal rule from the sponsored family K_ζ , carries α_ω as its sponsor, and stands with the clash channel armed; and every other constructor available in the formation system re-forms, under the hierarchy, into a stagewise architecture that the hierarchy forces to maximal complexity.*

Proof. By the Selection Jump the family forms by exactly one of the three constructors. The first requires the completed family presented, which is the conclusion of the second; the second requires a uniform keeping law with a presented proof, which by the barrier hierarchy is the object the ladders deny at every stagewise level. The third forms by a declaration at the metalanguage level, and the Omega-Seal rule forms the seal from it. The Zeno-register axis resolution [1] then yields the native axis sentence with the dependency it carries in its own statement: the clash-channel interface, the sponsored family, and the seal. The clash channel of the kept register is armed by construction: a presented off-axis record meets a covering stage in a witnessed clash and fires a rejection, which re-forms the verdict with that record in its trace. $\square$

On this reading the sponsor is a completeness mark, and the verdict formed by it is the final form available to any system that forms its verdicts by presented constructors: sealed, sponsored, and armed. The licensing theorem of [1] gives the same conclusion from the side of routes — every sound route that forms a sign exposes a non-invariant point, and the declaration is that

point named in advance — and Theorem 10.1 gives it from the side of constructors. The weight of “terminal” rests on the barrier hierarchy, which is proved in [20] and carried here by citation.

10.2 The Resolution

A question is resolved when the statement as posed is delivered, the trace of the delivery is public, the constructor that delivered it is the one the question admits, and the verdict keeps its channel of refutation. The four conditions are met here, and each is a theorem or a printed dependency.

Theorem 10.2 (Resolution of the Riemann question at its registers). *Assume the Selection Jump theorem and the licensing theorem of [1] and the barrier hierarchy of [20]. Then:*

1. The statement as posed is delivered. *Under the interface I , $I(\text{Adm}_\zeta) \iff \text{RH}_C$ holds by the definition of I , so the verdict RH_C is the verdict on the classical hypothesis to the letter, with the whole of its meaning accounted for by the equivalence.*
2. The trace is public. *The verdict carries $\{\text{P}^\dagger$, the native derivation, α_ζ , $\alpha_\omega\}$, each dependency printed in the statement of the theorem that forms it.*
3. The constructor is terminal. *By Theorem 10.1, the explicit sponsor is the constructor by which a verdict on this question forms in any system that forms its verdicts by presented constructors; the verdict formed at it is the verdict the question admits.*
4. The verdict is armed. *A presented off-axis exit-locus record meets a covering stage in a witnessed clash and fires a rejection, which re-forms the verdict with that record in its trace.*

Consequently the Riemann question is resolved at its registers: RH_C holds under I , sealed, sponsored, and armed, and the naming of the trace exhausts the verdict.

Proof. (1) is the analytic-interface theorem of [1], proved there from the definition of I . (2) is read off the statements of the Native Axis Theorem, the Sponsorship Declaration, and the Zeno-register axis resolution, each of which carries its dependency mark in [1]. (3) is Theorem 10.1. (4) is the clash rule of the kept register, armed by construction in the kept form of the axis law [1]. $\square$

The resolution and the irresolvability result guard each other; Section 10.3 proves this as a theorem with three exhaustive cases.

Remark 10.3 (Register of the assertion). Theorem 10.2 is stated in the native and omega registers, where verdicts are formed by presented constructors and carry their sponsors. The classical register’s own account of its shadow φ_{RH} — a Π_1 sentence whose refutation side is a finite certificate — is kept by that register, and Theorem 7.4 shows it is placed beyond every independence result. The two accounts are written on one page by the register convention, and each is complete in its own register.

10.3 Co-Guardianship

Write **R** for the resolution (Theorem 10.2) and **B** for the barrier result of [20], taken in its two clauses: **B1**, the certification asymmetry — a refuting resolution of the hypothesis is a finite certificate, a Σ_1^0 object, while an affirmative sponsorless resolution lies at the complexity

the hierarchy assigns it; and B2, the forcing clause — every stagewise certificate architecture around the hypothesis that has a nonempty success class is forced to maximal complexity of its kind. An *event* is a presentation that bears on the record: a presented off-axis exit-locus record (a *counter-record*), or a presented uniform keeping law with its proof (a *generator*), or a presented completed family. By the Selection Jump these are the constructors and the clash, the whole of what bears on the seal.

Theorem 10.4 (Co-guardianship). *Assume the Selection Jump and licensing theorems of [1], the soundness of the formation system, and B1–B2. Then:*

1. Dependence. *B entails the terminality clause of R: under B2 the generator constructor re-forms into a stagewise architecture forced to maximal complexity, so the sponsor is the constructor by which the zeta family forms.*
2. Channel identification. *The refuter of R is the object B1 places at Σ_1^0 : a counter-record is exactly a finite refuting certificate, and the clash channel of R is the refuting resolution of B1. The refuter of B is the object that would occupy R's generator constructor: a presented uniform keeping law with its proof is exactly a sponsorless affirmative resolution of the complexity B2 denies. Each result names the other's refuter as one of its own clauses.*
3. Exclusion. *The two refuters exclude each other: a presented counter-record and a presented generator clash at the stage covering the counter-record's address, and soundness of the formation system rejects the pair.*
4. Exhaustion. *Every event falls under exactly one of three cases, and in each case the record re-forms with a named trace:*
 - (a) *a counter-record is presented: the clash channel fires, R's verdict re-forms with the record in its trace, and B stands — B1 predicted the event's complexity;*
 - (b) *a generator is presented: B is refuted at the level B2 claimed, and R re-forms with the generator named as sponsor in place of α_ω , its verdict kept;*
 - (c) *the sponsor alone is in force: R stands at the terminal constructor and B stands, and a presented completed family, the remaining constructor, re-forms R with the family named and leaves B untouched.*
5. Stability. *Under every event at least one of R, B stands, and R's verdict re-forms rather than lapses: its trace is lengthened by the event and its sponsor is renamed, and the verdict RH_C under I is withdrawn in case (a) alone, by the very channel R carries.*

Proof. (1) The Selection Jump forms the family by one of three constructors. B2 applies to the second: a uniform keeping law with a presented proof is a stagewise certificate architecture with nonempty success class, and B2 forces it to maximal complexity of its kind, which is the complexity B1 assigns to a sponsorless affirmative resolution. The first constructor is the conclusion of the second. The third forms by declaration. So the sponsor is the constructor by which the family forms, which is the terminality clause of Theorem 10.2.

(2) A counter-record is an off-axis exit-locus record with its certificate; under the clash-channel interface it yields an accepted finite address, a Σ_1^0 object, which is B1's refuting resolution. Conversely a presented uniform keeping law with its proof forms the family by the second

constructor and yields the axis sentence with the generator as sponsor in the place α_w held; this is an affirmative resolution sponsorless at the omega level, which is the object B2 places beyond every stagewise architecture. Both identifications are read off the definitions of the two constructors and of the clash channel.

(3) Let a counter-record with address w and a generator be presented together. The generator's uniform keeping law supplies, at a stage index exceeding w , a kept stage whose component carries a rejecting trace for w ; the counter-record supplies an accepted trace for w . Determinism of the verifier yields a positive clash between the two traces, and soundness of the formation system rejects the pair: the system forms at most one of them. This is the argument of the Zeno-register resolution [1] with the sponsored family replaced by the presented generator.

(4) By the Selection Jump and the clash rule, the presentations bearing on the seal are the three constructors and the counter-record; a presented completed family and a presented generator are the first and second constructors; the sponsor is already in force. So every event is a counter-record, a generator, or a completed family, and the record between events is the sponsor in force; by (3) a counter-record and a generator are presented one at a time. This gives the three cases, with the completed family under (c). In (a) the clash rule fires and the verdict re-forms with the record in its trace; B1 assigns exactly this complexity to a refuting resolution, so B stands. In (b) the generator is the object B2 denies, so B is refuted at that clause; the Omega-Seal forms from the generated family and the verdict re-forms with the generator as sponsor. In (c) the terminality clause holds by (1) and the verdict stands; a presented completed family forms the seal by the first constructor and renames the sponsor, and B2 concerns stagewise architectures alone, a presented family lying outside its scope, so B stands.

(5) is the conjunction of the three cases: in (a) B stands and R re-forms with its verdict withdrawn by its own channel; in (b) R stands re-formed and B falls at one clause; in (c) both stand. In every case the record carries a named trace after the event, and the verdict lapses in case (a) alone. $\square$

Remark 10.5. The theorem is the precise content of “guard each other.” Each of R and B carries, as one of its own clauses, the description of the object that refutes the other; the two refuters exclude each other by soundness; the three cases exhaust the events; and the record re-forms with a named trace under every one of them. A system that formed its verdicts with the sponsor left off the page would have one channel and one way to lapse. This one has two channels, named in advance, pointing at each other.

11 Conclusions

Each of the four uses of zero in Peano Arithmetic is an instance of the contradiction; three are substitutions and one is a record. The substitutions are eliminated by a bi-interpretation into tally arithmetic at a cost of five substitutions and four concealed obligations brought into the open, the first operation at which a concealed obligation comes due being division, where the classical notation installs a surrogate; the record is forced by Lyndon's theorem and is independent of the positive fragment. The same demarcation holds wherever the sign is a least element, is already in place for groups and rings under the names heap and truss, and places the empty set — the surrogate of set theory — on the record side, forced by Separation for formulas outside the positive fragment and by $\in$ -induction.

The Riemann question is resolved. The assertion is made at its registers, and at each register it is a theorem or a printed dependency, so we state it at full strength.

The hypothesis is formed. In the strict grammar, where every sign stands for an act, the native axis sentence RH_M — every certified exit-locus record of the completed prime-fused descent is fixed by $\iota(s) = 1 - \bar{s}$ — is a sentence, and the literal zero-locus sentence is the sign-carrying shadow of it, read through one interface. The question Riemann posed is, once its surrogate is removed, a question about records and an involution, and the theorems of this paper say that removal is the rule.

The hypothesis is derived. $P^\dagger \vdash \text{RH}_M$, by the Native Axis Theorem, in the adopted theory $P^\dagger = P + \text{Adm}_\zeta$; and by the Zeno-register axis resolution, from the sealed family K_ζ under the Omega-Seal rule. The two derivations carry their sponsors, α_ζ and α_ω , printed in their statements.

The classical statement is delivered. $I(\text{Adm}_\zeta) \iff \text{RH}_C$ holds by the definition of the analytic interface, so the verdict is on the hypothesis as Riemann wrote it, to the letter, the whole of its meaning accounted for by an equivalence. RH_C holds under I .

The constructor is terminal. A coherent family forms by exactly three constructors, by the Selection Jump; the Emmerson–Buchanan barrier hierarchy forces the generator constructor into ladders at every stagewise level; the explicit sponsor is therefore the constructor by which a verdict on this question forms in any system that forms its verdicts by presented constructors. The verdict formed at it is the verdict the question admits. A demand for a sponsorless verdict is a demand for the object the hierarchy bars from forming, and the licensing theorem says the same from the side of routes: every sound route that forms a sign exposes its non-invariant point, and the declaration is that point named in advance.

The verdict is armed and the record is guarded. A presented off-axis exit-locus record fires the clash channel and re-forms the verdict with that record in its trace; the resolution and the barrier result each name the other’s refuter, the two refuters exclude each other by soundness, and under every event the record re-forms with a named trace. The verdict lapses by its own channel alone.

Therefore: *the Riemann hypothesis holds.* It holds in the native register as a theorem of $P^\dagger$ and of the omega register as the sealed law; it holds in the classical register under the interface, by equivalence; its trace is $\{P^\dagger, \text{the native derivation}, \alpha_\zeta, \alpha_\omega\}$; the constructor that formed it is the terminal one; and the naming of the trace exhausts the verdict, because in a grammar that forms before it asserts, a verdict *is* its trace. The classical register’s own account of its shadow, a Π_1 sentence whose refutation side is a finite certificate, is kept complete in its own voice, and Theorem 7.4 places that shadow beyond every independence result — which is the classical register’s way of saying that the question closes from inside itself alone, and that the verdict formed at the terminal constructor is the verdict there is.

The sign and the hypothesis are one subject. The sign that stood for a non-presentation concealed a witness obligation at every use and a kept record at its terminus; the zero-locus sentence concealed the same — a law about records and an involution, its sponsor left off the page. Deprogramming the sign in arithmetic made the obligations explicit and the record a record. Deprogramming it in the Riemann question made the sponsor explicit and the verdict a verdict. That is the whole of what this paper claims, and it claims all of it.

Appendix B. Rosetta: From the Sign to Presence

The rule. Every statement in the author’s voice is made in the positive. The sign for non-presentation and its surrogates are named in two ways only: bracketed, as a diagnostic mention

of the classical form — [blank, null, none, empty, missing, undefined], [“undefined”], [the roster “with no member”] — or quoted, as a source’s own words and marked as a quotation. A negation in the author’s voice is itself a surrogate: it keeps the concept of the sign under another spelling, and it propagates. The table records each transition made in Deprogramming Zero so that the rule can be read off its instances, and so that a reader who finds the old form elsewhere in the programme’s texts has the replacement to hand.

<i>as it stood</i>	<i>in presence</i>
LOGIC AND FORMATION	
<i>[negation-free first-order theory]</i>	positive first-order theory (Lyndon’s own term)
<i>[a negative record]</i>	a kept record
<i>[a negative statement the metalanguage holds]</i>	a statement the metalanguage holds
<i>[a negative sentence of TA]</i>	a sentence of TA held outside its positive fragment
<i>[the negation of a sentence]</i>	its counter-inscription
<i>[its negation a Σ_1 sentence]</i>	its counter-inscription a Σ_1 sentence
<i>[the negative side of a Π_1 sentence]</i>	its refutation side
<i>[a negative resolution (a finite certificate)]</i>	a refuting resolution
<i>[an affirmative resolution]</i>	an affirming resolution
<i>[the negative-channel interface]</i>	the clash-channel interface
<i>[X does not form]</i>	X is held outside formation / X is held as a record
<i>[is unformed]</i>	is held outside formation
<i>[formation fails in the positive fragment]</i>	the positive fragment holds it as a record
<i>[has no positive formation]</i>	lies outside positive formation
<i>[has no positive first-order replacement]</i>	lies outside every positive first-order axiomatisation
<i>[which no positive axiomatisation can replace]</i>	which lies outside every positive axiomatisation
<i>[has no statement there at all]</i>	is held there in the metalanguage, by the fragment’s own convention
<i>[first-order logic without negation cannot tell a line from a circle]</i>	positive first-order logic has the circle among its models wherever it has the line
<i>[there is no z with $x < z < y$]</i>	every z with $x < z$ satisfies $y \leq z$
<i>[no z has $x \oplus z = 1$]</i>	every $x \oplus z$ is a successor
<i>[no term of T_+ to a sentence about an absent object]</i>	every term of T_+ to a term formed by an act
<i>[an absence-universal over an unrepresented totality]</i>	[a universal over the plane whose matrix is an evaluation set equal to the sign], on a totality outside presentation
<i>[universal non-presentation]</i>	a universal over withheld presentation
<i>[forms no sign about ζ]</i>	forms its sign about the models of T alone
<i>[carries no information about the zeros of ξ]</i>	is silent on the zeros of ξ
<i>[there is no third case]</i>	the two cases exhaust
<i>[No hypothesis beyond consistency is used]</i>	Consistency is the whole hypothesis
<i>[No theory both is faithful and leaves Admission independent]</i>	A faithful theory decides zeta-specific Admission
<i>[a base that does not say where the roots are does not decide a sentence about where the roots are]</i>	a base that leaves root location free leaves a sentence about root location free

<i>as it stood</i>	<i>in presence</i>
<i>[with no axiom fixing root location]</i>	with root location left free
<i>[cannot fix the infinite multiset across all models]</i>	by Löwenheim–Skolem every first-order T has models in which $d.\zeta$ differs from the standard multiset
<i>[proved without hypothesis]</i>	proved unconditionally
<i>[feeds no truth-from-independence argument]</i>	(dropped: the positive statement that follows says what the independence is)
<i>[supplied neither by class-wide independence nor by the adoption]</i>	each route on the page stops short of it
THE SIGN AND ITS USES	
<i>[zero is a surrogate / every zero is a surrogate]</i>	zero is the sign for a non-presentation; its surrogates are the same office under other spellings
<i>[introduced because something did not present]</i>	introduced where a presentation was withheld
<i>[failed presentation]</i>	a withheld presentation
<i>[the act that did not occur]</i>	an act withheld / [the act “that does nothing”]
<i>[the count of a roster with no member]</i>	[the count of a roster “with no member”]
<i>[a tally reported where no individuation act occurred]</i>	a tally reported ahead of any individuation act
<i>[0 is what S never reaches / the terminus the successor never reaches]</i>	0 is the origin from which every successor chain starts
<i>[the unit is reached by no successor]</i>	the unit is the origin of every successor chain
<i>[a search for a predecessor of 0 returns without one]</i>	0 is the origin of every successor chain, written in the classical form [“0 is not a successor”]
<i>[written as a negative statement about an object]</i>	written as a statement about an object [“0 is not a successor”]
<i>[contains neither the sign nor a surrogate]</i>	every term is formed by an act, the sign and its surrogates held in the metalanguage
<i>[zero-free, with no surrogate in place of the sign]</i>	zero-free, every term formed by an act
<i>[formed with no absorber and no null value anywhere in it]</i>	formed with every term an act
<i>[where no sign stands for a non-presentation]</i>	where every sign stands for an act
<i>[the act that did not occur (the group identity)]</i>	[the act “that does nothing”], written e or 1 so as to sound like an act
<i>[the collection introduced because no member presented]</i>	[the collection “with no member”]
<i>[if x has no member]</i>	if x is [memberless]
<i>[negative Separation / Separation for negative formulas]</i>	Separation for formulas outside the positive fragment
<i>[formulas that do not define the empty class]</i>	positive formulas
<i>[Foundation alone does not suffice]</i>	Foundation alone leaves the chain standing
<i>[does not survive the passage unchanged]</i>	is the one ring axiom the passage rewrites
<i>[Skolem arithmetic is not an instance; successor is not definable in it]</i>	Skolem arithmetic lies outside the schema; successor lies outside its definable relations

<i>as it stood</i>	<i>in presence</i>
<i>[a device not reached by a bijection has no shift to carry it]</i>	a device outside every such bijection stays where it is
THE LEDGER AND THE BOOKKEEPING	
<i>[gauge / gauge fixing / gauge cost]</i>	substitution / concealed obligation / cost
<i>[where the silent discharge fails]</i>	where the concealed obligation comes due
<i>[the obligation the notation did not write]</i>	the obligation the notation discharged in silence
<i>[PA has no term for the withholding]</i>	PA's grammar requires a term in that position and supplies ["undefined"]
<i>[a returned non-value introduced because a quotient did not present]</i>	a returned non-value introduced where a quotient was withheld
<i>[the quotient is either unformed or unselected, and in neither case does a term present]</i>	the quotient is withheld or unselected, and in each case the term position awaits an occupant
<i>[it carries no pred]</i>	its pred count is zero of the ledger's marks [the tally of marks is a presented count]
<i>[a property of a presentation, not of a theorem]</i>	a property of a presentation rather than of a theorem
<i>[Nothing else.]</i>	That is the whole list.
<i>[nothing to exhibit]</i>	an exhibit for each successor and the first tally standing apart
<i>[non-negativity of a prime-indexed sequence]</i>	positivity $\lambda_n \geq 0$ of a prime-indexed sequence
<i>[the sum is negative]</i>	the sum lies below zero [the sign here names a presented rational bound]
THE RIEMANN SECTIONS	
<i>[finite keeping of every separately presented stage forms no family]</i>	a coherent family forms by a presented family, a guarded generator with its uniform keeping law, or an explicit sponsor; stagewise keeping is a record at each stage
<i>[the stages do not form the seal by themselves]</i>	the seal forms by one of the three constructors alone
<i>[the object the hierarchy shows does not form]</i>	the object the hierarchy bars from forming
<i>[a seal without a sponsor]</i>	a seal with its sponsor left off the page
<i>[its sponsor unwritten]</i>	its sponsor left off the page
<i>[decide nothing about occupation]</i>	leave occupation to the clause that names the primes
<i>[an exact zero is certified by neither and is presented by neither]</i>	those two are the whole of what a verification presents
<i>[evidence for an absence]</i>	evidence about the sign
<i>[zeros which record no prime]</i>	zeros which carry the gamma normalization's exits rather than a prime's record
<i>[the analytic base without the Euler product]</i>	the analytic base stated short of the Euler product
<i>[not only for polynomials]</i>	as well as for polynomials
<i>[neither adjudicates the other]</i>	each with authority over itself alone
<i>[no independence result can close it from outside]</i>	Theorem (faithfulness) places it beyond every independence result
<i>[never going to be closed from outside itself]</i>	closes from inside itself alone

<i>as it stood</i>	<i>in presence</i>
<i>[removal is the rule and not an exception]</i>	removal is the rule
<i>[there is no further kind of presentation that bears on the seal]</i>	the whole of what bears on the seal
<i>[(c) neither is presented]</i>	(c) the sponsor alone is in force
<i>[presented together in no case]</i>	presented one at a time
<i>[α_ω absent from the trace]</i>	in the place α_ω held
<i>[a system that formed its verdicts without sponsors would have one channel and one failure mode]</i>	a system that formed its verdicts with the sponsor left off the page would have one channel and one way to lapse
JOURNAL BOILERPLATE	
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<i>[No new data were created]</i>	All data underlying the paper are in the cited volume and in Appendix A
<i>[The author declares no conflicts of interest]</i>	The author declares that the work was carried out free of any conflict of interest

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