The Mirror Calculus

A Presence-Only Mathematical Language:
Formation, Descent, Geometry, and the
Register-Typed Resolution of the Riemann Question

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Abstract

This volume develops a mathematical notation in which every mark is kept under left–right reflection with every numeral formed by strokes [the numeral zero stands outside the inventory], and carries that notation from its grammar through geometry, algebra, analysis, and analytic number theory. Formation precedes assertion. The Mirror Calculus enacts presence-only formation: an object-language inscription forms only through a positive witness, trace, relation, transformation, roster, pairing, certificate, or explicit sponsorship event. Every object-language term has a sponsor, and a sponsor is a presentation. Numerical zero, [empty collections, null returns, default falsity], vacuous judgments, and [negative records inferred from a failed search] are analyzed as surrogate-zero devices: they make non-presentation participate as an object, value, or verdict.

The strict Mirror grammar has exactly twelve object constructors:

$$\mathsf{Bal},\ \mathsf{Row},\ \mathsf{Jux},\ \mathsf{Stk},\ \mathsf{Up},\ \mathsf{Dn},\ \mathsf{Lk},\ \mathsf{Ovl},\ \mathsf{Adh},\ \mathsf{Box},\ \mathsf{OBox},\ \mathsf{Frac}.$$

Strict object atoms are fixed by horizontal reflection. Reflection reverses the arguments of Bal, Row, and Jux, and acts pointwise on the remaining constructors. The resulting involution, well-formedness preservation, and renderer equivariance have a full structural proof. A smaller implementation fragment G_{K} is represented by the accompanying Lean source; the present edition keeps that fragment and the whole language distinct. In particular, the current implementation datatype contains a variant named `blank`. The kernel artifact therefore concerns implementation control syntax, with the stricter separation staged: the discipline adopted here, under which every layer's marks are emissions of formed terms, a mark for non-emission.

Blank stands apart from a term denoting [emptiness] and apart from a metasyntactic control either: every carrier at every layer carries a formed term, and withheld emission is an event of the metalanguage. $\mathrm{Eval}(t) \downarrow e$ records successful emission and $\mathrm{Eval}(t) \uparrow$ records the act's non-performance. A one-sided balance or an [empty enclosure] is thereby excluded from the language by the plenary-emission totality, and the judgment is the whole report of a withholding.

The Riemann question is resolved at explicitly separated registers, each verdict carrying its sponsor on its face. The native axis sentence RH_{M} is a sentence of $\mathrm{Sent}(\mathsf{G}_{\mathrm{P}})$; the literal zero-locus sentence $\mathsf{RH}_{\mathrm{C}}^{(0)}$ is formed in the classical register alone. The adopted theory $\mathsf{P}^{\dagger} = \mathsf{P} + \mathrm{Adm}_{\zeta}$ derives the native sentence:

$$\mathsf{P}^{\dagger} \vdash \mathsf{RH}_{\mathrm{M}}.$$

Over the declared base, class-wide Admission and its counter-inscription are each witnessed by a finite model in exact rationals: a relative independence theorem with both sides exhibited, and the fork at which Adm_{ζ} is adopted, sponsored by α_{ζ}. The witnessed semantics sponsors a coherent omega-kept family as a completion postulate, declared as such. Under the named classical interface,

$$I(\mathrm{Adm}_{\zeta}) \quad \Longleftrightarrow \quad \mathsf{RH}_{\mathrm{C}},$$

proved from the definition of the interface, so that RH_{C} holds under the interface with the sponsored adoption printed on the verdict. Independence is here an affirmative method: carried by a proof-reflecting translation to a sound classical theory, Σ_1-completeness converts it into arithmetic truth, and the analytic equivalence converts truth into RH_{C}. An explicit polynomial witness carries every reflection and functional-equation symmetry together with an off-axis quartet, and so measures exactly what Adm_{ζ} supplies beyond symmetry. The work supplies a presence foundation, a strict mirror grammar, a typed root algebra with its dagger reading, a kernel-certified fragment with its computational record, a research program recorded at exact strength, an external blind-reading audit of the object language, the native derivation, the class-wide model theorem, and the exactly priced analytic interface through which the classical statement is read.

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Preface

This book presents the Mirror Calculus: a mathematical notation in which every mark is kept under left–right reflection and the numeral every term is formed by an act [the sign stands outside the inventory], carried from its grammar through geometry, algebra, analysis, and analytic number theory. It is the full Mirror Calculus volume of the three-paper *Against Zero / Counting Back from Infinity / Mirror Calculus* program: the first paper defends the thesis that formation precedes assertion and analyzes surrogate-zero devices by role; the second adopts the descent laws and develops native mathematics relative to them; the present volume gives the language, its algebra and geometry, its plates, and the register-typed resolution of the Riemann question. The object language is the protagonist throughout; every classical appearance in the book is an interpretation the interface prices.

The tour is this. The presence foundation opens the book, and the descent mathematics follows it. The strict mirror language is then given exactly: codex, twelve-constructor grammar, well-formedness, reflection metatheory, and the scope of its formal verification. The plate language comes next as current mathematics, closing with the caption-blind decoding audit — the language delivered to a reader, the prose standing aside. The typed algebra and its geometry follow: the failure of the first junction model and the typed successor, the reading group that folds the page, roots and resolvents, the dagger reading, and positive analytic counting. The cone-chain applications lead into the Riemann architecture, where the controlling chain of the front matter is proved component by component; the explicit formula and the computational record follow; and the book closes with the research directions and their engineering consequences.

Conventions. Classical results imported into proofs are named at the point of use. The metalanguage is ordinary mathematical English and may discuss zero, emptiness, nonformation, classical logic, and standard mathematics. The strict object language is narrower: every term of it stands for a presentation, and a term whose purpose is to reify non-presentation.

What is new here. Four things, and it is worth separating them from what is imported. First, a reflection-equivariant presence-only grammar with a closed atom inventory, twelve constructors, and a kernel-certified fragment: a formal language whose every object-level term stands for a presentation, carried far enough to state and check real mathematics. Second, the resolution of the Riemann question at its own registers: in the presence grammar the native axis sentence is well formed and the adopted theory derives it, while the conventional zero-locus sentence belongs to the classical register, where it converts the withholding of a presence-valued evaluation into equality with a numerical object; the classical statement is read off the native law through one interface at a price stated exactly, and presence-only formation, the grammar in which this is carried out, is argued for in the first chapter and adopted as the grammar of record throughout. Third, the register discipline itself: an assertion-record schema under which every consequential claim names its sponsor, its stratum, and the interface priced for any transfer, applied to a hard subject in full. Fourth, the surrogate-zero taxonomy and its engineering transfers, which a fully classical reader

may adopt entire.

What sponsors what. Each result of this book is read off its sponsor. $\mathsf{P}^{\dagger} \vdash \mathsf{RH}_{\mathrm{M}}$ is a derivation. The relative independence of class-wide Admission from the base is a theorem with both sides exhibited by finite models. The interface equivalence $I(\mathrm{Adm}_{\zeta}) \Longleftrightarrow \mathsf{RH}_{\mathrm{C}}$ is proved from the definition of the interface. Adm_{ζ} itself is present in $\mathsf{P}^{\dagger}$ by declaration, sponsored by α_{ζ}, at a fork the independence theorem proves inhabited on both sides; and under the interface RH_{C} accordingly holds, with the verdict carrying that sponsor on its face. The symmetry no-go formalizes, for the same architecture, a constraint the classical reader already meets. The residue is named and located: faithful reference to the completed prime-fused descent, which the volume shows carries the whole of the remaining classical difficulty.

Claim and Register Convention

Every claim in this book is typed by the register in which it is made. Five registers are used: the grammatical register $\mathsf{Reg}_{\mathrm{Gra}}$, in which formation in the presence grammar G_{P} is judged; the native object register $\mathsf{Reg}_{\mathrm{Obj}}$, in which the descent laws operate and the adopted theory $\mathsf{P}^{\dagger}$ derives; the omega register Reg_{ω}, in which formation under the registered Omega-Seal rule is judged; the relative register $\mathsf{Reg}_{\mathrm{Rel}}$, for derivability and independence over the unextended base P and for statements conditional on a declared adoption; and the classical register $\mathsf{Reg}_{\mathrm{Cla}}$, entered only through the named interface I. Statements migrate between registers through named interfaces alone: a transfer names its interface, and the interface prices it.

Four kinds of claim occur: stipulations, which declare grammar or adoption; theorems, proved in a stated register; computations, exact or carried out at stated truncation and precision; and interpretations, which read one register's content in another and carry theorem strength only componentwise. Where a machine certificate applies, its scope is stated once, where it is used.

Theorem 0.1 (Register noncollapse)**.** *The five registers each stand in their own authority: for each pair there is a sentence lawful in one and unformable or retyped in the other. Every clause of this convention keeps a classical verdict into a native one, or a native proof into a classical one, through the named interface alone.*

The controlling architecture

The object language is sovereign in this book. Its sentences are formed by the twelve-constructor grammar; its theorems are proved in the registers the grammar sustains; and every classical appearance in the volume is an interpretation whose price a named interface states. The mathematical spine is the following chain, proved component by component. Formation:

$$\mathsf{RH}_{\mathrm{C}}^{(0)} \notin \mathrm{Sent}(\mathsf{G}_{\mathrm{P}}), \qquad \mathsf{RH}_{\mathrm{M}} \in \mathrm{Sent}(\mathsf{G}_{\mathrm{P}}).$$

Native relative independence:

$$\mathsf{P} \nvdash \mathrm{Adm}_{\mathrm{class}}, \qquad \mathsf{P} \nvdash \mathsf{CounterAdm}_{\mathrm{class}}.$$

Explicit foundational selection:

$$\mathsf{P}^{\dagger} = \mathsf{P} + \mathrm{Adm}_{\zeta}.$$

The resolution, a native proof:

$$\mathsf{P}^{\dagger} \vdash \mathsf{RH}_{\mathrm{M}}.$$

The boundary is native as well: the polynomial witness

$$F(s) = \left((s-\tfrac{1}{2})^2 - a^2\right)\left((s-\tfrac{1}{2})^2 - \bar{a}^{\,2}\right), \qquad a = \tfrac{3}{10} + 7i,$$

satisfies every reflection and functional-equation symmetry the tent group imposes while carrying an off-axis quartet, and it recurs as a central theorem below.

The interface, priced. Through the named analytic interpretation I,

$$I(\mathrm{Adm}_\zeta) \Longleftrightarrow \mathsf{RH}_{\mathrm{C}}, \qquad \text{so} \qquad I \models \mathsf{P}^\dagger \implies \mathsf{RH}_{\mathrm{C}}, \qquad \neg\mathsf{RH}_{\mathrm{C}} \implies I \not\models \mathsf{P}^\dagger.$$

The classical statement enters here and here alone: as the priced shadow of the adopted native law.

Contents

Part I

Presence Foundation

Chapter 1

Formation Before Assertion

1.1 The foundational thesis

Formation precedes assertion. A strict inscription enters the object language through a positive formation event: a witness, trace, relation, transformation, nonempty roster, pairing, certificate, or explicit sponsorship. The formation event supplies the inscription and its provenance together.

The thesis is stated positively. The strict language records what a scene presents and how the presentation forms. Evaluation judgments and search control remain in the metalanguage, and implementation control codes remain in the implementation stratum. They stay short of acquire object status merely because a software datatype or explanatory sentence needs to discuss them, and the language of record is exhausted by formed terms, with a counterpart to any of them standing outside it.

Whenever a formed symbol is assigned the semantic office of representing non-existence, the notation embeds a category conflict: the mark's formation certifies presentation while its assigned role asks it to stand for a withheld presentation. Presence-only formation resolves the conflict by retaining the formed mark, when one exists, as implementation or diagnostic control rather than promoting it to a strict semantic object.

This is a claim about type and formation, ahead of any Boolean verdict declaring one mathematical tradition permitted and another forbidden. Classical mathematics and the presence calculus are distinct formal regimes. Their relation is established theorem by theorem through named interfaces.

1.2 The positive formation of one

Unity may be positively presented. A natural scene may exhibit a stable singular organization; a counting procedure individuates that presentation and carries it into tally grammar as one. The phrase *imposed unity* names this individuation operation. It stands apart from mean that nature is incapable of presenting singular form.

The Descent interpretation gives a second realization. In the sweeping geometry, unity forms through the balance between an infinitesimal length carried through an infinite angle and an infinite length carried through an infinitesimal angle. Written schematically in the metalanguage,

$$(\mathrm{d}\ell)\,\Theta_{\infty} \quad \mathsf{Bal} \quad \ell_{\infty}\,(\mathrm{d}\Theta).$$

The balance is an act of formation [rather than a discovery of a context-free numeral floating outside formation. It is a positive relation whose sweep, scene, and trace form a unit carrier.

Counting therefore requires:

1. a presented scene;
2. a positive individuation relation;
3. a trace establishing the unit carrier;
4. a procedure that repeats or compares such carriers;
5. a roster on which the tally is performed.

The numeral one records the first formed unity of that procedure. Genealogical numeration retains the sweep history by which the unity formed.

Root grammar and semantic office

Root grammar governs more than the printed BNF. It governs the semantic office assigned to a formed mark.

A metalanguage may quote or diagnose a privative expression. The root-grammatical violation occurs when that expression is made to perform object-forming or inference-sponsoring work.

Whenever a presented symbol is assigned the meaning "non-existence," root grammar receives both a positive inscription and a semantic instruction cancelling presentation. This is a root-grammatical contradiction, even where a classical model consistently manipulates the resulting sign.

The strict discipline therefore asks, at every use:

1. Which positive event formed the mark?
2. Which positive relation gives it semantic content?
3. Does the mark carry presented content, or is it functioning as renderer or search control?
4. Has a control-layer event been promoted into a semantic object?

The surrogate-zero test is determined by role, ahead of vocabulary. "Zero," "empty," "null," "none," and privative prose are equally subject to the test whenever their meanings are applied.

1.3 Surrogate-zero roles

device	reifying role	presence-only replacement
numerical zero	a tally returned ahead of any individuated member	a tally forms only over a nonempty roster and therefore begins at one
empty collection	[a roster "with no presented member"]	a roster forms from sponsored members, one at least; its formation requires that one sponsored entry

null return	a returned value standing in for a withheld value	successful emission is recorded by Eval(t) $\downarrow$ e; withheld emission is the metalanguage judgment Eval(t) $\uparrow$
default falsity	a refuting verdict inferred from a search that closed short of an affirmative verdict	a contrary requires its own positive witness or a completed coverage certificate
vacuous judgment	a verdict over an unoccupied domain	the object-language judgment forms exactly when a domain roster or a separately priced classical interface is supplied
[negative record from a failed search]	a record of exclusion inferred from non-record	exclusion is its own record, formed by positive coverage and completion traces
blank enclosure	a visible frame made to denote [an empty object]	a diagnostic renderer display about withheld emission, outside the strict sentence language

A positive contrary is its own inscribed act, with its own sponsor. Rejection, exclusion, failure, incompatibility, and fault are lawful object-language contents when independently witnessed. An instrument fault report is a positive event. A completed search certificate is a positive event. A witnessed clash between two formed inscriptions is a positive event. What is barred is the conversion of silence into any of them with the sponsor's place standing open.

1.4 Three statuses, two truth values

The notation distinguishes:

1. a positively presented assertion;
2. a positively presented contrary or rejection, with its own witness;
3. withheld inscription: the pair of statuses standing open.

Status (iii) stands apart from the truth values. It stands apart from any object that propagates through connectives, a null member of a result type, or an "unknown" value. It is the metalanguage observation that the pair of statuses stands open: that of the first two inscriptions formed. A three-valued logic reifies the third status as a participant in operations. TPN holds it in the metalanguage.

1.5 Classical mathematics and transfer

The foundational thesis leaves classical mathematics standing meaningless or useless. It denies automatic transfer.

A positively interpreted fragment of a classical structure may be used when the interpretation presents its elements, domains, and operations. A theorem then transfers only after a separate argument shows that its statement and proof remain inside that fragment. Every use of [an empty auxiliary structure], choice over [an unpresented] family, excluded middle over [unformed sentences], or [a null element] requires a named interface. The price may be worth paying; it may not be hidden.

The theorems of this volume stop short of establishing that all classical mathematics has such a transfer. Conversely, every stalled automatic transfer is represented as a refutation of the classical theorem in its own register.

Chapter 2

Transcendental Presence Notation

2.1 Primitive positive sorts

TPN is the program's trace-bearing decision language. Its primitive sorts are:

Witness. A presented individual, event, or performed act.

Scene. A context of presentation: an instrument, procedure, epoch, laboratory, database scope, or other positive setting.

Trace. A record of the procedure by which a witness, relation, or transformation was presented.

Inscription. A formed object-language sign carrying its sponsor.

Nonempty roster. A sequence of one or more presented inscriptions together with apartness or individuation records.

Tally. A count over a formed nonempty roster.

Pairing. A presented correspondence between witnesses or scenes.

Positive transformation. A trace-bearing act carrying presented input to presented output.

Clash record. A record of two formed inscriptions and a separately witnessed incompatibility between them.

Certificate. A distinguished inscription recording a completed procedure, its scope, and its coverage traces.

Sponsorship. A positive adoption event by which a foundational law or completion postulate is entered into the adopted theory.

2.2 Formation rules

A witness forms an inscription only with a scene and trace:

$$\frac{\mathsf{Witness}(w) \qquad \mathsf{Scene}(\sigma) \qquad \mathsf{Trace}(\pi : w \text{ presented in } \sigma)}{\mathsf{Inscribe}(\ulcorner w \urcorner; \pi)}.$$

A roster requires one or more entries and positive individuation:

$$\frac{\mathsf{Inscribe}(t_1;\pi_1)\ \cdots\ \mathsf{Inscribe}(t_k;\pi_k) \qquad \mathsf{Sep}(t_i,t_j;\sigma_{ij})\ (i<j) \qquad k\geq 1}{\mathsf{Roster}(\langle t_1 \frown \cdots \frown t_k\rangle;\pi_1 * \cdots * \pi_k)}.$$

A tally forms only from a roster:

$$\frac{\mathsf{Roster}(R;\pi)}{\mathsf{Tally}(R)\geq 1}.$$

Thus counting begins at one as a consequence of the adopted roster formation rule.

A certificate records positive completion:

$$\frac{\mathsf{Procedure}(p) \qquad \mathsf{Scope}(S) \qquad \mathsf{Completed}(p,S;\tau) \qquad \mathsf{Coverage}(p,S;\kappa)}{\mathsf{Cert}(p;S;\tau,\kappa)}.$$

2.3 Direct clash and hypothesis-discharging rejection

Direct clash recording and hypothesis-discharging rejection are different acts.

A direct clash requires two formed inscriptions and an independent witness of incompatibility:

$$\frac{\mathsf{Inscribe}(t;\pi) \qquad \mathsf{Inscribe}(t';\pi') \qquad \mathsf{Clash}(t,t';\kappa)}{\mathsf{ClashRecord}(t,t';\pi,\pi',\kappa)}.$$

A rejection under a hypothesis requires derivations on both sides of a witnessed clash:

$$\frac{\mathsf{Assume}(h;\alpha) \qquad \mathsf{Derive}(t \mid h;\pi) \qquad \mathsf{Derive}(t' \mid h;\pi') \qquad \mathsf{Clash}(t,t';\kappa)}{\mathsf{Reject}(h;\alpha,\pi,\pi',\kappa)}.$$

A term held outside formation is a metalanguage judgment, standing apart from a clash. Indirect reasoning remains available, but it must terminate in two positively formed conclusions together with a positive incompatibility witness.

2.4 Sponsorship

Foundational laws carry their sponsors on the page. They enter the adopted theory through a positive adoption event:

$$\frac{\mathsf{Sponsor}(\alpha,\varphi)}{\mathsf{Assert}(\varphi;\alpha)}.$$

The event stands apart from a proof of φ from weaker laws. It records which theory has been selected and who or what performed the selection.

This rule governs both Native Admission and the omega completion used later. It is the principal device by which the volume separates adopted law from theorem.

2.5 Scene composition

Scenes compose only through presented pairings. If witnesses in scenes σ and τ are identified by a trace-bearing pairing, the pairing licenses transport between those scenes. Every scene is built.

The frequently used idea that local scenes assemble into a scene-colimit is an *interpretation*. A formal colimit model is not supplied by the current calculus. Accordingly, this volume does not cite scene-colimit language as a theorem of TPN.

2.6 Engineering transfers

The formation rule has direct engineering consequences.

Databases

A record's coverage, and that alone, licenses an exclusion. Under an open-world reading, a row yet to present warrants a coverage question. A closed-world inference becomes presence-lawful only when a positive completeness certificate establishes that the relevant relation, sources, and time range were covered. This sharpens the familiar distinction between incomplete information and recorded falsity in database theory [16, 65, 54].

A semantic schema should therefore store events and relations that formed. A renderer or query layer may withhold a field where a value is yet to was emitted; the control path stays control. A domain value called NULL is formed only by a named interface, and a classical export that requires one is that interface.

AI and missing data

A dislike is a recorded dislike event; a rating is an emitted rating; a feature value is an emitted value; the unobserved cell has status (iii). Implicit-feedback methods that assign a low-confidence signal to unobserved cells install a modeling device [40], and presence-only reporting keeps the device labelled as one.

Padding tokens, masks, sentinel codes, and withheld emissions belong to the implementation-control layer. They guide tensor shape and control flow, and a semantic report records only what was emitted: the machinery layer's marks reach it, when they do, through a named interface.

Safety and decision systems

A consequential decision should be a positive record such as

$$\mathsf{Approved}(\text{request}; \text{sponsor}; \text{trace}),$$

$$\mathsf{Rejected}(\text{request}; \text{clash}; \text{trace}),$$

or

$$\mathsf{ReviewScheduled}(\text{request}; \text{trigger}; \text{slot}).$$

A rejection forms from a clash; an approval withheld holds at status (iii). A detector that remains silent leaves ["no hazard"] outside formation. The presence replacement is a readiness certificate recording scope, test traces, instrument traces, credentials, and coverage. Such provenance accords with the positive-event orientation of the W3C PROV model [63].

Part II

Descent Mathematics

Chapter 3

The Adopted Descent Foundation

The laws in this chapter are adopted formation laws. Their status is not empirical, and they are not consequences of the strict grammar. The no-zero result proved from them is therefore relative to them.

Adopted Formation Law 3.1 (Plenum)**.** *The Plenum is the adopted horizon of undifferentiated total presence on which sweeps act. It is not an empty base, a null element, or a rosterable object with freely available members.*

Adopted Formation Law 3.2 (Sweep)**.** *A sweep is a positive act of differentiation across a presented horizon. Every sweep carries a trace identifying the act, its scene, and its relation to prior sweeps.*

Adopted Formation Law 3.3 (Precipitation)**.** *A residue forms when interference among sweeps concentrates positive support according to the adopted presentation condition. A residue is therefore a formation event with a genealogy, not an untraced existential posit.*

Adopted Formation Law 3.4 (Genealogical Numeration)**.** *A native numeral records the formation history of a residue: descent degree, parent sweeps, branch, and order of formation. Equal classical magnitude does not erase distinct genealogies.*

Adopted Formation Law 3.5 (Fractal Continuation)**.** *A presented formation pattern may continue to further stages through its own trace-bearing generator. The law does not identify stagewise availability with a completed total family.*

Adopted Formation Law 3.6 (Sweeping Law)**.** *Every native formation is a sweep or a trace-bearing composite of sweeps. The trace of a composite retains the traces of its constituents.*

3.1 Relative no-zero theorem

Theorem 3.7 (No null tally under the adopted descent laws)**.** *In the native tally sort generated by the adopted Plenum, Sweep, Precipitation, Genealogical Numeration, Fractal Continuation, and Sweeping laws, every term's tally runs over an inhabited roster; a tally over an unoccupied roster.*

Proof. The base formation rules require a positive witness or precipitation event. Every composite rule preserves at least the witnesses and traces of its premises. Roster formation requires at least one entry. An induction over derivations therefore shows that every formed tally has an inhabited sponsoring roster. A [null tally] stands outside derivation under these rules. □

Remark 3.8. The theorem is relative to the adopted laws. It stands apart from a proof that classical zero is contradictory, impossible in every foundation, or outside classical models.

3.2 Genealogy and arithmetic

Juxtaposition of genealogies and nesting of one genealogy through another supply candidate readings of addition and multiplication. This volume uses those readings only where a corresponding evaluation identity is displayed or proved. It does not assert a general arithmetic representation theorem for all genealogical numerals.

The n-wave reading—that a degree-n residue family is realized by n-fold interference—is likewise an interpretation. No complete formal n-wave model is included in the current artifact set.

3.3 Classical completion

Let S be a semigroup whose every element moves every element. Its free identity adjunction is

$$S^1 = S \sqcup \{e\},$$

with multiplication or addition extended so that e is an identity and with exactly those relations imposed.

For the positive additive tally semigroup,

$$(\mathbb{Z}_{>0}, +)^1 \cong (\mathbb{N}, +, 0).$$

Classical zero is recovered here by an explicit adjunction. A genealogy-forgetting map that sends a formation history to its positive magnitude leaves the identity to the priced adjunction. Consequently, any classical passage requiring zero must name the completion or adjunction; the priced completion alone carries it; projection alone.

Part III

The Strict Mirror Language

Chapter 4

The Symbol Codex

Everything below is normative. Layer 1 is semantic ground; Layer 2 is the retired linear system (documented for the record and for linearization); Layer 3 is the final strict alphabet with stroke-level constructions; Layer 4 the layout constructors; Layer 5 the derived notational conventions; Layer 6 the metalanguage.

4.1 Layer 1 — semantic ground

Strict-symmetry axiom. Every mark is carried to itself by left–right reflection, enforced by construction (stroke specifications), not by font.

Mirror reading. $M(S)$ = order reversal composed with glyph-wise reflection; the semantic requirement is $[\![M(S)]\!] = [\![S]\!]$ where $[\![\cdot]\!]$ (kerned double brackets).

Non-inscription. The [empty region] stands apart from the symbols; it is the absence of one. Every classical "= 0" is answered by blank paper: annihilation, coincident bounds, closed-manifold accumulation, discriminant balancing blank, the refused basepoint of the indefinite integral.

Blank is not a name (the non-euphemism clause). Blank is not zero renamed; it differs in syntactic category, not vocabulary. Zero is a term: it takes properties, enters sets, receives operations, and anchors the identity and absorber laws. A blank region emits no glyph, and every term's office is presence: each names a formed presentation, the offices are exhausted by the constructor inventory, and whatever is present in syntax is present in emission — a withholding is the renderer's non-performance of the act, recorded by the judgment $\mathrm{Eval}(t) \uparrow$ and by it alone. What zero names in the classical register is answered here by an act's non-performance, never by a mark. *Eliminability criterion:* every well-formed occurrence of Blank paraphrases, meaning-preserved, into presence-talk (coincidence, annihilation, unsatisfiability, keeping); an occurrence that cannot be so paraphrased is smuggled zero and is illegal. *Inexpressibility criterion:* classical sentences in which zero is essential ("zero is even"; the identity law) have no translation — they are held outside formulation, their truth values with them. Zero is ineliminable and generative of laws; Blank is eliminable and generative of prohibitions. At the level of the classical model the two are co-referential; the system reforms the grammar, not the referent, and whether that reform is discovery is exactly the Faithfulness Question. *Corollary (the nominalization ban).* Blank stands outside plurality, outside membership, gradation, index, and duration: "the exit loci," "a set of exit

loci," "blank-depth," "blank at level n" are all illegal prose — reifications of absence, the zero-move committed in the metalanguage. All such structure attaches lawfully to *sentences* (inscriptions) and *certifiers* (presences): one writes "true Π_1 sentence (arithmetical hierarchy, written without the oracle superscript throughout)s," "sentences undecided by the level-n certifier," never their blank-nominal shadows. Discovered as the clause's fourth successful audit: the eliminability test convicted a metatheoretic exposition that had granted absence a census, and every convicted phrase paraphrased losslessly into sentence-and-certifier form — which is the clause functioning as designed: a euphemism cannot be convicted; a typed prohibition can. *Sanctioned forms.* Each survives the eliminability test: (i) the metalanguage judgment Eval(t) $\uparrow$, which records withheld emission as a statement about the evaluation relation, never as a term; (ii) the predicative idiom *balances blank*, which predicates non-inscription of an evaluation exactly as "is consistent" predicates of a theory — a statement about emission, mirroring no drawn form; (iii) *blank-typed* as an adjective on sentences and truth-conditions; (iv) *blank* as an ordinary adjective on physical presences (blank paper, a blank page); (v) sentential nominals of the predicate — *non-inscription* — which attach to processes and sentences exactly as "consistency" and "non-halting" do. (vi) the copular and elliptical predicatives of the same idiom — "is blank," "blank $\Leftrightarrow$," "ascending or blank" — which predicate non-inscription of an evaluation exactly as *balances blank* does. (vii) for laws, the positive-act forms are preferred and used natively: *keeping law* where the classical register writes kept, *mirror-fixed* and *carried to itself* where it writes kept-under-reflection; "keeping" remains licensable as a sentential nominal but is reserved for classical-apparatus names, flagged as imports. A law is an act performed at every stage, not a stasis. (viii) every position is an occupation: a drawn layout presents formed content at each of its positions and the page is exhausted by emissions of formed terms, so a one-sided balance or empty enclosure is excluded from Sent(G_P) by that totality — the exclusion a fired rejection sponsored by the plenary-emission clause, in the grammar and equally in the drawn layout (WF6, Definition 5.11); the renderer's withholding is an event of the metalanguage, recorded by the judgment. Three plate emissions violating this law (a blank fused as an addend) were detected by the blind-reading experiment, whose independent reader — offered a vacant slot in an additive frame — introduced a null object to fill it: the exact category collapse the typing charge indicts, induced by our own layout. The emissions are repaired; the law is enacted; the experiment stands as the register's first external audit instrument. All count-noun uses are abolished: the parameters at which a descent balances blank are named what they always were — **exit loci**, presences, markable, orbit-bearing; the ledger over them is the **exit ledger**; the Witness–Blank theorem is renamed the **Witness-Typing theorem**. The reform sharpens the Riemann chapters rather than weakening them: the hypothesis concerns exit loci — parameters of the strip — and only the misnamed noun was ever absent-shaped. *The logic of the volume (judgment forms).* The metalogic here is verificationist, in the lineage the Honesty clause names: there are no truth-values as objects — the Boolean pair is the zero–one pair in disguise — and a sentence does not *have* a value. There are three judgment forms, each an act with an inscription attached: a sentence is *inscribed* (a derivation exhibited), *counter-inscribed* (a refutation exhibited), or *kept* (its channel maintained stage by stage). Excluded middle accordingly stands apart: a law of this logic but a classical import, named where used — in the classical equivalences, and in the dichotomy clause of the Keeping Theorem, whose appeal to soundness and Σ_1-completeness is classical-side reasoning about the transfer. Assertion, in this logic, is the exhibition of an inscription. *Refusals are events.* Every "cannot" and "unwritable" in this volume is certified positively: by a fired rejection — the engine prints its refusal text, an inscription — or by an exhibited counter-

term, drawn and checked. Negation here is never a name for an absent object; it is always cashed as a presence that does the blocking. *The audit obeys the typing.* Universal negatives over open domains — "no element of the kind anywhere" — admit no finite certificate and are blank-typed: they may be kept, stage by verified stage, never concluded. This applies to the volume's own purity claims: what is certified is the ledger of convictions and discharges; whether further instances remain is a channel held open, in exactly the posture this volume holds toward its one open axiom. Precedent: $\neg\exists$ asserts absence without naming an object; Blank is the refused reification of the negated existential.

WF6 and vinculum canonicality — discovery record. The prediction experiment exposed two forms admitted by an earlier grammar stratum: Blank used as an additive operand and an inscribed unit denominator. Definitive WF6: Blank may occur only as a balance side or as the content of Box/OBox; it is prohibited in every other constructor position. The former emissions are preserved only as the evidence by which the specification gap was discovered.

Torsor ground. The number line is an affine line: positions and oriented changes, no origin. Orientation exists and lives on the vertical axis; what the classical register calls sign is read off it.

Balance form. Equations are written with both sides inscribed as presences; no canonical right-hand side, no "$= 0$" normalization.

1D Collapse (design lemma). Under the stated parsing and layout hypotheses together with the strict axiom a one-line string can express only commutative operations and symmetric relations; hence the notation is necessarily two-dimensional, with exactly two lawful carriers of asymmetry: the vertical axis, and adhesion.

4.2 Layer 2 — the retired chiral system, linearized

Achiral glyphs (self-converse denotations): $+ \cdot = \leftrightarrow \forall \pi$, the vertical stroke, and the provisional digit family. Chiral pairs, each mirroring to its converse:

pair	denotation	converse reading
angle brackets $\langle\ \rangle$	grouping	grouping
left/right filled triangles	oriented change "from–to"	"to–from"
lower-corner triangles	division (a by b)	(b under a)
$\nearrow$ / $\nwarrow$	power (base–exponent)	(exponent–base)
half-moon anchors	numeral units-position	converse significance
$<$ / $>$	order	converse order
$\rightarrow$ / $\leftarrow$	implication	converse implication
tail-arrows	one-sided approach (from below)	(from above)
lens brackets	oriented integral scope	reversed orientation
lollipop pair	positional "below"	converse
turnstiles $\vdash$ / $\dashv$	judgment (departing)	(arriving)

Additional achiral operators of that layer: $\curlywedge$ (minimum), a vertical-symmetric existential, three-dot digit-three (freeing Δ for the differential). *Retirement:* the pairs violate the strict axiom individually; the layer is retained solely as a typeable linearization of Layer-2 terms, and its two glyph classes are recovered in the final theory as the trivial and sign isotypes of the flat reading group.

4.3 Layer 3 — the strict atoms, stroke by stroke

All coordinates are in units of the atom half-size s (engine value $s = 11\,\text{pt}$ at scale 1), origin at the atom's center, x rightward, y upward. Every construction is symmetric under $x \mapsto -x$ by inspection. "Pip" = filled dot of radius $0.35s$; "dot" (operator) = filled dot of radius $0.16s$; "rank dot" = filled dot of radius $0.3s$.

Digits (bijective values one to ten)

value	name	construction
1	stroke	line $(0, -s)$–$(0, s)$
2	chevron	lines $(-0.7s, -s)$–$(0, s)$ and $(0, s)$–$(0.7s, -s)$
3	three bars	horizontals at $y = -0.8s, 0, 0.8s$, each $(-0.8s, y)$–$(0.8s, y)$
4	four pips	pips at $(\pm 0.55s, \pm 0.55s)$
5	quincunx	pips at $(\pm 0.6s, \pm 0.6s)$ and $(0, 0)$
6	six pips	pips at $(\pm 0.5s, \{0.7s, 0, -0.7s\})$
7	branched stem	line $(0, -s)$–$(0, s)$; arms $(0, \pm 0.15s)$–$(\pm 0.75s, \pm s)$, all four
8	double circle	circles radius $0.52s$ centered $(0, \pm 0.52s)$
9	nine pips	pips at $(\{-0.65, 0, 0.65\}s, \{-0.65, 0, 0.65\}s)$
10	cross	lines $(-0.7s, -s)$–$(0.7s, s)$ and $(-0.7s, s)$–$(0.7s, -s)$

Letters (variables)

Y	stem $(0, -s)$–$(0, 0)$; arms $(0, 0)$–$(\pm 0.7s, s)$
A	legs $(\mp 0.7s, -s)$–$(0, s)$; bar $(-0.38s, -0.15s)$–$(0.38s, -0.15s)$
H	verticals at $x = \pm 0.55s$ full height; crossbar at $y = 0$
T	top bar $(-0.7s, s)$–$(0.7s, s)$; stem $(0, s)$–$(0, -s)$
U	verticals $(\pm 0.55s, s)$–$(\pm 0.55s, -0.3s)$; bottom arc (semicircular arc closing below, 180° extent)
V	lines $(\mp 0.7s, s)$–$(0, -s)$
M	verticals at $x = \pm 0.7s$; inner strokes meeting at $(0, -0.1s)$
W	four strokes: $(\pm 0.8s, s)$–$(\pm 0.4s, -s)$–$(0, 0.2s)$
Ω	open arc $-60°$ start, 300° extent, in box $(\pm 0.6s, -0.45s..0.75s)$; feet $(\pm 0.3s.. \pm 0.62s, -0.55s)$

Extension rule: further variables by symmetric diacritics (dot, diaeresis, circumflex) glued above these; asymmetric Latin letters are illegal.

Operator and mark atoms

$+$	crossbars $(\pm 0.7s, 0)$ and $(0, \pm 0.7s)$; commutative addition
$\cdot$	operator dot; commutative multiplication
$=$	horizontals at $y = \pm 0.3s$; symmetric relation
$\oplus$ (fuse)	circle radius s with internal $+$ of half-arm $0.55s$; presence fusion (commutative semigroup, no neutral)
$\cap$ (cap)	upper semicircular arc, legs to $y = -0.7s$; meet-junction (commutative by WF3 — the grammar forces the algebra)
rank dot	glued above a digit: one dot $= \times 10$, two dots $= \times 100$, etc.
Δ	triangle, apex $(0, s)$, base $(\pm 0.85s, -s)$; the differential mark, glued above its variable
∞	circles radius $0.42s$ centered $(\pm 0.42s, 0)$; the indeterminate-extent mark
sector	upper half-disc radius s with diameter chord; the positive sector, carries a glued $+$
up / dn	vertical arrow marks (stem $1.5s$, symmetric heads); bare orientation atoms
dots	three pips at $(\{-0.55, 0, 0.55\}s, 0)$; ellipsis / residue marks
disk / rim	circle radius $1.5s$, solid / dashed; region and boundary
rayV / rayH	long stroke, vertical $(0, \pm 2.2s)$ / horizontal $(\pm 2.2s, 0)$; sweep carriers

4.4 Layer 4 — constructors and layout law

Universal metrics: gap $G = 8\,\mathrm{pt} \times \mathrm{scale}$; adhesion marks render at $0.55\times$ host scale; box padding $1.4G$; Up/Dn connector $2.6s$; Lk connector $1.8s$.

constructor	layout	reflection action
$\mathsf{Row}_\circ(t_1;t_2)$	horizontal through operator atom $\circ$; **WF3**: $\circ$ must denote commutative/symmetric content	swaps arguments (harmless by WF3)
$\mathsf{Jux}(t_1;t_2)$	operator-free juxtaposition (clusters, free pairs); commutative by fiat	swaps arguments
$\mathsf{Stk}(t;b)$	t above b, common axis	pointwise
$\mathsf{Up/Dn}(t;b)$	stack joined by oriented vertical arrow; the oriented change "from b up to t" / "from t down to b"	pointwise (orientation is vertical: mirror-fixed)
$\mathsf{Lk}(t;b)$	plain link; order-by-height	pointwise
$\mathsf{Ovl}(a;b)$	superposition at a common center (sweep crossings)	pointwise
$\mathsf{Adh}_{\uparrow/\downarrow}(h;m)$	mark m (atom *or term*) glued above/below host h; **WF2**: rank dots on digits; Δ on variables; exponents and half-powers above; stage/index marks below	pointwise on host and mark
$\mathsf{Box}(t)$	rounded enclosure; definite scope	pointwise
$\mathsf{OBox}(t)$	enclosure open at the top; **WF4**: only for semantically indeterminate content (infinity carriers, missing bounds)	pointwise
$\mathsf{Frac}(n;d)$	vinculum stack; division and quotients	pointwise
Blank	emits no element; the licensed non-inscription slot	fixed

WF1: all layout axes vertical. **Mirror theorem** (structural induction): every well-formed term denotes the same proposition as its mirror; the only argument-permuting constructors are Row/Jux, tamed by commutativity. **Machine check**: render term and reflected term to primitive multisets (lines, circles, discs, arcs, rects); the term passes iff the coordinate-reflection of the first matches the second under bipartite pairing with tolerance 0.05 pt (arcs compared with $\theta \mapsto 180^\circ - (\theta + \text{extent})$).

4.5 Layer 5 — derived notational conventions

Numerals. Bijective base ten; a number is a cluster (Jux) of rank-marked digits, order-free; e.g. thirty-two $= \{3^\bullet, 2\}$ in either order; one hundred $= \{9^\bullet, 10\}$; $1024 = \{10^{\bullet\bullet}, 2^\bullet, 4\}$.

Oriented change. $\mathsf{Up}(q;p)$ = "from p up to q"; its magnitude is achiral; annihilation $\mathsf{Up}(p;p) + \mathsf{Dn}(p;p)$ balances blank.

Roots. A square root is the half-power fraction glued above (Adh with mark $\mathsf{Frac}(1;2)$). "The" root is never inscribable: the pair is achiral; a root with exhibited orientation carries an up/dn orientation mark — Galois conjugation is arrow reversal. Solving an equation is licensed chirality descent; Vieta (coefficients) is the achiral projection.

Calculus. Differential: Δ glued above the variable. Derivative: $\mathsf{Frac}(\Delta W;\Delta T)$ — adhesion over vinculum, both vertical; existence = mirror-coherence of the vertical approach pair. Definite integral: Box with bounds stacked (target above, source below), integrand with glued Δ inside; degenerate bounds $\Rightarrow$ Blank. Indefinite integral: OBox; the result ends "+ Blank"

— the classical $+C$ is the refused basepoint. FTC: the box balances the Up-change of the antiderivative.

Blanks (the non-inscription family). annihilation; coincident bounds; closed-manifold accumulation; disjoint junction; discriminant balancing blank (repeated roots; symmetric configurations sit on their own branch loci); zero deficit (the fourth right corner); the refused $+C$; the unsatisfiable ω-equation on the flat page.

Carrier tower. Which roots a form can hold is fixed by its reading group: page — sign roots; corner — adds cube roots of the turn (E-doublet); smooth cone — all turns: the cone is, as carrier language, angularly complete (interpretation; the fundamental theorem of algebra is an import, not derived here) (the FTA reading).

4.6 Layer 6 — metalanguage symbols

G, μ, M, $[\![\cdot]\!]$	grammar; glyph reflection; mirror reading; semantics
$\mathsf{Sw}, \mathsf{Res}, \mathcal{K}$	MJA sorts: sweeps, residues, costs (all zero-free)
$\oplus, \cap, \kappa, \delta, \varepsilon, \pi^+, \mathrm{tr}$	fusion; junction; cost; defect (non-inscribed iff transverse; recoverability iff the defect balances blank); non-inscription (meta-name); positive-sector projection; genealogy trace (free — Chasles lives in evaluation only)
$t, \kappa = \sin\alpha, \alpha, \gamma, R, h$	sector fraction; sine; semi-vertical angle; seam angle ($\sin\gamma = t_1h_2 + t_2h_1$: the circle group law); slant; height ($h^2 = (1-t)(1+t)$: deficit times abundance)
$\nu_{i,n}$	Bessel order; limit-circle iff $\nu interiortotheunitinterval$; rank thresholds $\kappa = \sqrt{(4n^2+1)/5}$
e_k, disc	symmetric functions ($e_1 = 1$ always: the circle as trace); discriminant (the inscribed distinction)
A_k strata	fold / cusp / swallowtail: balanced di-cone; equilateral tri-form; four quarters — symmetry depth = multiplicity at the exit locus
G (reading groups)	$\mathbb{Z}/2$ (page), C_{2v} (tent), C_{3v} (corner), $O(2)$ (cone), $O(3)$; carriers = kept coordinates; chiralities = nontrivial irreps
$U(2)$, $\Lambda = U\mathrm{B}U^*$	seam/apex self-adjoint families; the junction dictionary (spend = extension parameter/degree)

Terminological convention

The following five words are used throughout in these senses only. An *assertion* is an inscription entered into $\mathrm{Sent}(\mathsf{G_P})$ together with its sponsor. A *judgment* is a metalanguage act about formation or evaluation — $\vdash$-statements, $\mathrm{Eval}(t)\uparrow$, formation and well-formedness judgments; a judgment stands apart from the truth-bearers. A *value* is the output of evaluation, an element of $\mathbb{C}^\times$. A *verdict* is the recorded outcome of a sponsored decision. *Truth* is reserved for satisfaction in a named classical interpretation: the predicate is two-place, as in *true in the standard arithmetic interpretation*, and within a section that has already fixed the interpretation the qualifier may be elided. The phrase *truth-value* names a classical notion and appears below only where that notion is being denied of some object of this calculus; the Boolean pair is the zero–one pair and is named as such. Unqualified truth stands outside the terms of this volume, and every claim here is made in a register.

Remark 4.1 (One sentence for the whole package). A language in which every mark survives the mirror is forced into two dimensions; its regular fragment is the dagger-kept content; its exit loci are the classical exit loci; folded, it acquires larger reading groups whose irreducible representations are exactly the kinds of roots it can carry; and completed to the cone it carries them all.

Chapter 5

The Strict Grammar

5.1 Atoms

Definition 5.1 (Atom inventory)**.** The atom set Σ consists of finitely many marks, each drawn symmetric about its own vertical axis:

digits	$\mathsf{d}_1, \ldots, \mathsf{d}_{10}$ (stroke, chevron, three bars, pips 4–6, branched stem, double circle, pips 9, cross)
letters	$\mathsf{A}, \mathsf{H}, \mathsf{M}, \mathsf{T}, \mathsf{U}, \mathsf{V}, \mathsf{W}, \mathsf{Y}, \Omega$, extended by symmetric diacritics
operators	$+$, $\cdot$, $=$, $\oplus$, $\cap$ (all denote commutative content; see WF3)
marks	rank dot, Δ (differential), ∞, the sector glyphs
connectors	plain vertical stroke, upward arrowhead, downward arrowhead

The axiom of the system is that every atom is fixed by left–right reflection; this is enforced by construction (each atom is specified as a symmetric stroke set), not assumed of a font.

5.1.1 Constructors

Definition 5.2 (Layout constructors)**.** Terms are generated from atoms by exactly seven constructors:

$$t \ ::= a \in \Sigma \ \mid \ \mathsf{Stk}(t_1; t_2) \ \mid \ \mathsf{Up}(t_1; t_2) \ \mid \ \mathsf{Dn}(t_1; t_2) \ \mid \ \mathsf{Adh}_{\uparrow/\downarrow}(t; m)$$
$$\mid \ \mathsf{Box}(t) \ \mid \ \mathsf{OBox}(t) \ \mid \ \mathsf{Row}_{\circ}(t_1; t_2) \ \mid \ \mathsf{Frac}(t_1; t_2)$$

with intended layout: $\mathsf{Stk}(t_1; t_2)$ places t_1 above t_2 on a common axis; $\mathsf{Up}(;)$ and $\mathsf{Dn}(;)$ are stacks joined by an oriented vertical connector; $\mathsf{Adh}_{\uparrow}(t; m)$ glues mark m directly above the head atom of t (resp. below for $\downarrow$); $\mathsf{Box}(t)$ encloses; $\mathsf{OBox}(t)$ encloses with the top edge left open; $\mathsf{Row}_{\circ}(t_1; t_2)$ juxtaposes horizontally through an operator atom $\circ$; $\mathsf{Frac}(t_1; t_2)$ is the vinculum stack. (Nine constructors are listed; Up, Dn and $\mathsf{Box}, \mathsf{OBox}$ are orientation/openness variants of two, giving seven constructor families.)

Definition 5.3 (Well-formedness)**.** WF1 Every layout axis of every subterm coincides with a vertical line; $\mathsf{Stk}(;)$, $\mathsf{Up}(;)$, $\mathsf{Dn}(;)$, $\mathsf{Frac}(;)$ center their arguments on a common axis.

WF2 $\mathsf{Adh}(;)$ may glue only atoms of the mark class; a rank dot may glue only to a digit; Δ only to a letter or sector; exponent and half-power fractions glue above; stage and index marks glue below.

WF3 (*The commutativity discipline.*) $\mathsf{Row}_\circ(t_1; t_2)$ is well-formed only when $\circ$ denotes a commutative operation or a symmetric relation. All non-commutative content is expressed by $\mathsf{Up}(;)$, $\mathsf{Dn}(;)$, $\mathsf{Frac}(;)$, $\mathsf{Adh}(;)$, or openness — never by horizontal order.

WF4 $\mathsf{OBox}(t)$ is well-formed only when the unterminated edge of t is semantically indeterminate (an infinity carrier); openness is the inscription of indeterminacy and may not decorate determinate content.

5.1.2 Reflection and the mirror theorem

Definition 5.4 (Reflection action)**.** ρ acts on terms by: $\rho(a) = a$ for atoms; $\rho(\mathsf{Stk}(t_1; t_2)) = \mathsf{Stk}(\rho t_1; \rho t_2)$, likewise for $\mathsf{Up}(;)$, $\mathsf{Dn}(;)$, $\mathsf{Frac}(;)$, $\mathsf{Box}()$, $\mathsf{OBox}()$, and $\mathsf{Adh}_v(t; m)$ pointwise; $\rho(\mathsf{Row}_\circ(t_1; t_2)) = \mathsf{Row}_\circ(\rho t_2; \rho t_1)$.

Theorem 5.5 (Mirror keeping — derived)**.** *For every well-formed term t, $[\![\rho t]\!] = [\![t]\!]$.*

Proof. Structural induction. Atoms: fixed by ρ and drawn self-symmetric, so the base case is the atom axiom. Vertical constructors ($\mathsf{Stk}(;)$, $\mathsf{Up}(;)$, $\mathsf{Dn}(;)$, $\mathsf{Frac}(;)$, $\mathsf{Box}()$, $\mathsf{OBox}()$, $\mathsf{Adh}(;)$): ρ acts pointwise on arguments and preserves the constructor, so the inductive hypothesis closes the case — reflection cannot permute vertically encoded data. The only constructor that permutes arguments is $\mathsf{Row}(;)$, and WF3 restricts its operator slot to commutative/symmetric denotations, so $[\![\mathsf{Row}_\circ(\rho t_2; \rho t_1)]\!] = [\![\rho t_2]\!] \circ [\![\rho t_1]\!] = [\![t_2]\!] \circ [\![t_1]\!] = [\![t_1]\!] \circ [\![t_2]\!]$. □

Remark 5.6. The theorem shows why the earlier plates were *sound* but not yet a notation: soundness lived in ad hoc layout choices that happened to respect WF1–WF4. The grammar internalizes those choices; from here on, a figure is correct iff it parses.

5.1.3 The plates as terms

Proposition 5.7 (Compositionality check — derived)**.** *Each previously drawn figure is a term of* G*. Representative parses:*

- *Oriented change "from one up to eight":* $\mathsf{Up}(\mathsf{d}_8; \mathsf{d}_1)$.
- *Thirty-two: the multiset* $\{\mathsf{Adh}_\uparrow(\mathsf{d}_3; \textit{rank dot}),\ \mathsf{d}_2\}$ *(a* $\mathsf{Row}_+(;)$*-free cluster; cluster juxtaposition is commutative juxtaposition and passes WF3).*
- *The definite integral of Plate 6:* $\mathsf{Row}_=(\mathsf{Stk}(\mathsf{d}_2; \mathsf{Stk}(\mathsf{Box}(\mathsf{Row}_\cdot(\mathsf{Row}_\cdot(\mathsf{d}_3; \mathsf{Adh}_\uparrow(\mathsf{Y}; \mathsf{d}_2)); \mathsf{Adh}_\uparrow(\mathsf{Y}; \Delta))); \mathsf{d}_1)); \mathsf{Row}_=($
- *The unterminated ray of Plate 9:* $\mathsf{OBox}(\mathsf{Adh}_\downarrow(\mathsf{ray}; \infty))$*, legal by WF4.*
- *The residue of Plate 9:* $\mathsf{Box}(\mathsf{Row}_\cap(\mathsf{Row}_\oplus(\mathsf{Adh}_\downarrow(\mathsf{A}; \bullet); \mathsf{Adh}_\downarrow(\mathsf{V}; \bullet)); \mathsf{Adh}_\uparrow(\mathsf{sector}^+; +)))$*, where* sector^+ *names the drawn upper-sector atom.*

No figure requires a constructor outside Definition 5.2; the artistic appearance of the plates was presentation of terms, and the same terms regenerate the same figures mechanically.

5.2 Exactly twelve object constructors

The strict object grammar has exactly twelve constructors over the strict atom set Σ:

$$\mathsf{Bal}, \mathsf{Row}, \mathsf{Jux}, \mathsf{Stk}, \mathsf{Up}, \mathsf{Dn}, \mathsf{Lk}, \mathsf{Ovl}, \mathsf{Adh}, \mathsf{Box}, \mathsf{OBox}, \mathsf{Frac}.$$

Blank is outside this count because Blank is not an object constructor.

Definition 5.8 (Strict terms)**.** The sort Term is generated by

$$\begin{aligned} t ::= a \mid \mathsf{Bal}(t,t) \mid \mathsf{Row}_\circ(t,t) \mid \mathsf{Jux}(t,t) \mid \mathsf{Stk}(t,t) \\ \mid \mathsf{Up}(t,t) \mid \mathsf{Dn}(t,t) \mid \mathsf{Lk}(t,t) \mid \mathsf{Ovl}(t,t) \\ \mid \mathsf{Adh}_v(t,m) \mid \mathsf{Box}(t) \mid \mathsf{OBox}(t) \mid \mathsf{Frac}(t,t), \end{aligned}$$

where $a \in \Sigma$, $\circ$ belongs to the admitted commutative/symmetric operator class, v is an admitted vertical adhesion position, and m is a well-typed mark term.

Atoms are generators, not constructors in the twelve-constructor count.

5.3 Constructor table

constructor	layout and typing	reflection action
$\mathsf{Bal}(l,r)$	two formed sides separated by the symmetric balance relation	$\mathsf{Bal}(\rho r, \rho l)$
$\mathsf{Row}_\circ(l,r)$	horizontal operator row; $\circ$ must denote commutative content or a symmetric relation	$\mathsf{Row}_\circ(\rho r, \rho l)$
$\mathsf{Jux}(l,r)$	operator-free multiset-style juxtaposition; order carries no denotation	$\mathsf{Jux}(\rho r, \rho l)$
$\mathsf{Stk}(t,b)$	t above b, with a common vertical axis	$\mathsf{Stk}(\rho t, \rho b)$
$\mathsf{Up}(t,b)$	vertical stack with an upward-oriented connector	$\mathsf{Up}(\rho t, \rho b)$
$\mathsf{Dn}(t,b)$	vertical stack with a downward-oriented connector	$\mathsf{Dn}(\rho t, \rho b)$
$\mathsf{Lk}(t,b)$	plain vertical link between presented terms	$\mathsf{Lk}(\rho t, \rho b)$
$\mathsf{Ovl}(l,r)$	overlay at a common center	$\mathsf{Ovl}(\rho l, \rho r)$
$\mathsf{Adh}_v(h,m)$	typed mark m adhered above or below host h	$\mathsf{Adh}_v(\rho h, \rho m)$
$\mathsf{Box}(t)$	closed enclosure of formed content	$\mathsf{Box}(\rho t)$
$\mathsf{OBox}(t)$	open enclosure of formed but semantically indeterminate content	$\mathsf{OBox}(\rho t)$
$\mathsf{Frac}(n,d)$	vertical vinculum stack, with formed numerator and denominator	$\mathsf{Frac}(\rho n, \rho d)$

Only Bal, Row, and Jux reverse their arguments. Every strict atom is fixed. The remaining constructors are carried pointwise under horizontal reflection.

5.4 Preterms and withheld emission

Definition 5.9 (Emission)**.** Emission is an act: a renderer emits the drawn form of a strict term, and the drawn page is exhausted by such acts. The act's non-performance is an event of the metalanguage, recorded by the judgment of Definition 5.10; the record is the judgment itself, and refusal to draw is the renderer's fired rejection, certified by that judgment as its inscription.

Definition 5.10 (Evaluation judgments)**.** Successful emission is written

$$\mathrm{Eval}(t) \downarrow e.$$

Withheld emission is written

$$\mathrm{Eval}(t) \uparrow .$$

The second expression is a metalanguage judgment about the evaluation relation. It stands apart from asserting that evaluation returned a special object.

Arithmetic, membership, parity, magnitude, orientation, duration, index, and genealogy attach to formed terms, and the operand, mark, host, numerator, denominator, roster, and enclosure positions are each occupied by formed terms — the occupancy exhausts the positions. Where a classical renderer prints a placeholder, this one performs the judgment instead, and the judgment is the whole of the record.

5.5 Well-formedness WF1–WF6

Definition 5.11 (Normative well-formedness)**.** The following six rules are the unique assertion-of-record well-formedness discipline.

WF1: fixed atoms and vertical axes. Every strict atom belongs to the closed reflection-fixed registry. Every constructor's principal layout axis is vertical. Renderer codes outside the registry do not become strict atoms.

WF2: typed adhesion. The second child of Adh must be a mark term from the closed mark grammar. Rank dots adhere only to digits; differential marks adhere only to admitted variable or sector hosts; exponent, stage, and index marks occupy their declared vertical positions.

WF3: horizontal invariance. A Row is admitted only for a commutative operation or symmetric relation. A Jux denotes an unordered rank-labelled cluster or another explicitly swap-invariant juxtaposition. A Bal denotes a symmetric balance relation. Horizontal order by itself carries no asymmetric content.

WF4: open-enclosure discipline. OBox(t) is admitted only when t is a formed term whose extent, bound, or continuation is semantically indeterminate under a declared interpretation. An object-level OBox always encloses a formed term; a withheld evaluation is an event of the metalanguage, recorded by the judgment, and the plenary-emission totality of WF6 cashes the exclusion of the visually empty enclosure.

WF5: numeral canonicality. A numeral contains exactly one digit at each occupied rank, no rank is repeated, no intermediate rank is omitted, every digit has value one through ten, and a unit denominator is not displayed. Each positive integer consequently has one canonical rank-labelled carrier.

WF6: plenary emission. Every drawn mark is the emission of a formed term, and every constructor position in a drawn layout is occupied by the emission of its formed immediate subterm: a balance presents two formed sides, an enclosure presents formed content, and the page is exhausted by such emissions. Emission is an act performed on formed terms; where the act goes unperformed, the metalanguage judgment $\mathrm{Eval}(t)\uparrow$ records the event. *Corollary, cashed by this clause as the blocking presence:* a one-sided balance or an empty enclosure is thereby excluded from $\mathrm{Sent}(\mathsf{G_P})$, in the grammar and equally in the drawn layout — a fired rejection in the sense of the refusals-are-events discipline, sponsored by the totality above.

5.6 Numeral canonicality

In bijective base ten, a canonical numeral has a finite rank roster

$$\{(r, d_r) : 0 \le r \le k,\ 1 \le d_r \le 10\},$$

with every rank $0, \ldots, k$ represented exactly once, and value

$$\sum_{r=0}^{k} d_r 10^r.$$

The roster is displayed through Jux; reflection may reverse the display order, but rank marks retain the value. For example, thirty-two is the cluster consisting of a rank-one digit three and a rank-zero digit two. One hundred is represented by rank-one digit nine and rank-zero digit ten. No placeholder is required.

Proposition 5.12 (Canonical carrier)**.** *Every positive integer has exactly one well-formed bijective-base-ten numeral under WF5.*

Proof. Existence and uniqueness are the usual division-with-remainder proof for bijective numeration, using remainders in $\{1, \ldots, 10\}$. If an ordinary remainder is zero, the preceding quotient is reduced by one and digit ten is used. WF5 records each resulting rank exactly once and excludes all alternative carriers with repeated, omitted, or unit-denominator ranks. □

5.7 Reflection

Definition 5.13 (Strict reflection)**.** Reflection $\rho : \mathsf{Term} \to \mathsf{Term}$ fixes every atom and is defined recursively by

$$\rho(\mathsf{Bal}(l, r)) = \mathsf{Bal}(\rho r, \rho l),$$
$$\rho(\mathsf{Row}_\circ(l, r)) = \mathsf{Row}_\circ(\rho r, \rho l),$$
$$\rho(\mathsf{Jux}(l, r)) = \mathsf{Jux}(\rho r, \rho l),$$

and pointwise on $\mathsf{Stk}, \mathsf{Up}, \mathsf{Dn}, \mathsf{Lk}, \mathsf{Ovl}, \mathsf{Adh}, \mathsf{Box}, \mathsf{OBox}, \mathsf{Frac}$.

Theorem 5.14 (Reflection involution)**.** *For every strict term t,*

$$\rho(\rho t) = t.$$

Proof. Structural induction on t. Atoms are fixed. In the Bal, Row, and Jux cases, two applications of reflection reverse the arguments twice and the inductive hypotheses restore the children. Every remaining constructor is preserved pointwise, so the inductive hypotheses close those cases. □

Theorem 5.15 (Well-formedness preservation)**.** *If t is well formed under WF1–WF6, then ρt is well formed.*

Proof. Proceed by structural induction. WF1 is preserved because strict atoms are fixed and horizontal reflection preserves vertical axes. WF2 is preserved because host and mark sorts are preserved recursively and the vertical adhesion position is unchanged. WF3 is preserved because the three argument-reversing constructors are restricted to swap-invariant content. WF4 is preserved because horizontal reflection does not change whether a formed enclosure is open or semantically indeterminate. WF5 is preserved because the rank-labelled numeral roster is unchanged as a roster. WF6 is preserved vacuously on strict terms because no control preterm occurs in either t or ρt. □

5.8 Renderer equivariance

Let $L(t)$ denote the ideal renderer emission as a finite structured family of primitives with exact coordinates, and let μ denote coordinate reflection $x \mapsto -x$ on those primitives.

Theorem 5.16 (Mirror renderer theorem)**.** *For every strict well-formed term t,*

$$L(\rho t) = \mu(L(t)).$$

Proof. The proof is structural. The atom case follows from the stroke registry. For $\mathsf{Bal}, \mathsf{Row}, \mathsf{Jux}$, horizontal reflection exchanges the child placement boxes, exactly matching the recursive argument reversal. For $\mathsf{Stk}, \mathsf{Up}, \mathsf{Dn}, \mathsf{Lk}, \mathsf{Adh}, \mathsf{Frac}$, reflection preserves vertical order and acts on each child. Overlay reflects each superposed primitive. Closed and open enclosure frames are themselves symmetric and carry the reflected child. These are all twelve constructor cases. □

Corollary 5.17 (Conditional semantic keeping)**.** *Suppose a semantic interpretation assigns symmetric balance to* Bal*, commutative content to each admitted* Row*, multiset semantics to* Jux*, and reflection-compatible meanings to the remaining constructors. Then*

$$[\![\rho t]\!] = [\![t]\!]$$

for every well-formed t.

Proof. Structural induction using the displayed interpretation hypotheses. The result is conditional on the semantic dictionary; renderer equivariance alone is not a proof of an arbitrary denotational interpretation. □

5.9 Formal verification

The reflection metatheorems proved in this chapter apply to the full twelve-constructor grammar G_{P}. The accompanying Lean development formalizes the implementation fragment G_{K}, whose constructors are

`atom`, `dig`, `blank`, `row`, `bal`, `jux`, `ovl`, `adh`, `box`, `obox`, `lk`, `frac`.

For this fragment the development proves reflection involutivity, preservation of its implemented well-formedness predicate, and equivariance of its emission algebra.

The implementation constructor `blank` is an artifact of the implementation grammar G_{K} alone; the strict discipline of record carries no counterpart to it at any layer, and it is not interpreted as a term of the strict object language. The full grammar theorem is therefore the structural theorem proved above, while the kernel certificate applies to the explicitly represented fragment G_{K}. Toolchain information and source checksums are supplied with the accompanying artifacts.

Part IV

The Plate Language

Chapter 6

Plate Grammar and Reading Rules

The plate corpus uses ordinary TikZ primitives embedded directly in this source. The presence of those primitives makes the figures reproducible by TeX, but it does not prove that they were regenerated from the current renderer or checked in the final environment.

The original corpus was divided into:

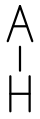
1. ten founding machine plates;
2. thirty folded and proof plates;
3. continued plates F31–F56.

The following source preserves their drawings and captions under the historical control notice.

Chapter 7

The Machine Edition: Original Ten Plates

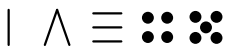
Plate 1 — carriers of asymmetry



order by height; the greater stands higher.

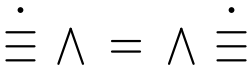
oriented change, from eight down to one.

Plate 2 — presence-only numerals



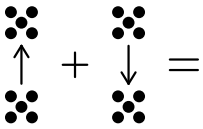
digits one through five.

digits six through ten.



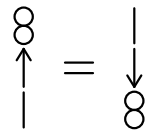
thirty-two, displayed in either horizontal order.

one hundred, represented by nine tens and ten.

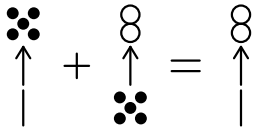


Diagnostic display: annihilation balances blank. Under the current discipline no drawn form reports withheld emission; the plate is retained as historical record (WF6).

Plate 3 — the torsor



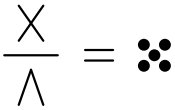
a change and its converse reading agree.



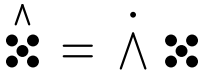
Chasles composition of oriented changes.

Plate 4 — balance quadratic

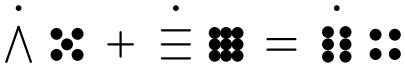
squares and ten roots balance thirty-nine.



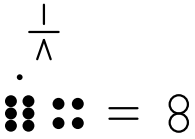
half the roots.



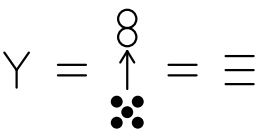
square it; twenty-five.



add the number; sixty-four.

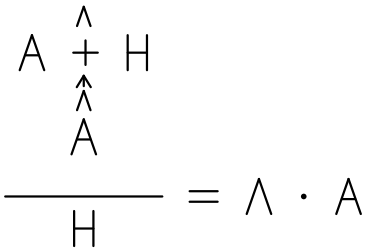


the half-power extracts a root.

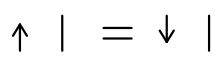


Y is the change from five upward to eight, giving three.

Plate 5 — mirror coherence

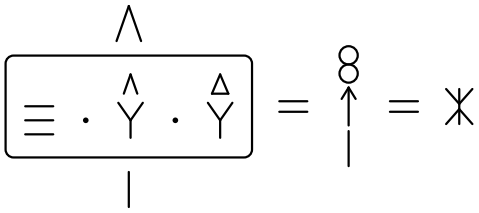


a quotient represented through vertical approaches.

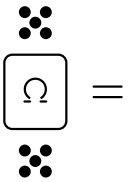


a visibly incoherent balance.

Plate 6 — integration

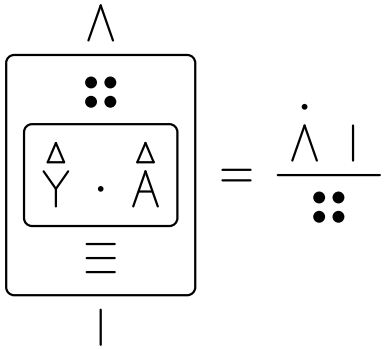


accumulate three Y-squares from one to two; the result is the change from one to eight, namely seven.



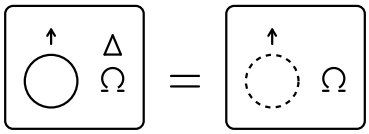
Diagnostic display: coincident bounds produce withheld emission. The current strict grammar classifies the empty side as a renderer diagnostic.

Plate 7 — Fubini

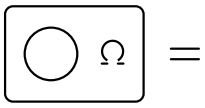


nested accumulation and its mirror, evaluated as twenty-one over four in the displayed finite example. General interchange still requires the applicable Tonelli or Fubini hypotheses.

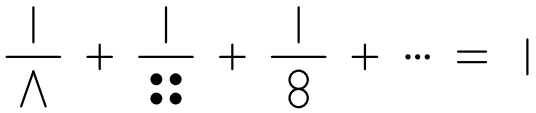
Plate 8 — Stokes and Zeno



disk and rim with orientation represented by a vertical mark.



Diagnostic display: a closed surface with no displayed rim contribution. The empty side is not a strict object-language term.



Zeno's finite remainders and partial sums balance the unit in the displayed finite pattern.

Plate 9 — descent

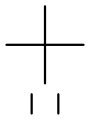
a line of indeterminate extent in an open frame.

transverse sweep carriers.

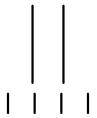
a proposed precipitated residue.

descent from the proposed plenum through staged residues.

Plate X — junction and genealogy

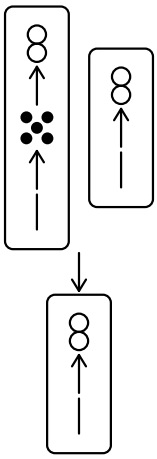


transverse configuration with a proposed minimal cost tally.



tangential configuration with a proposed surcharge. The surcharge law is superseded.

disjoint carriers; the partial junction is undefined.

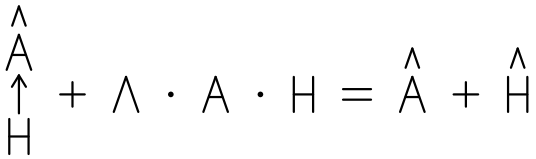


free syntactic traces above and an evaluated Chasles-style change below.

Chapter 8

The Folded Plates

Plate F1 — the tent: the dagger made isometry



a tent realization of primitive-level reflection. The current theorem is renderer equivariance; physical folding is an interpretation.

Plate F2 — three sheets and the cone limit

a wedge term on three faces and on a rotational cone. Distance from an apex is a classical invariant carrier; the apex itself is not removed by the strict grammar.

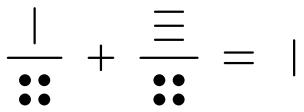
Plate F3 — the reading-group theorem

Historical caption-only plate. The page, tent, corner, cone, and space were assigned the groups

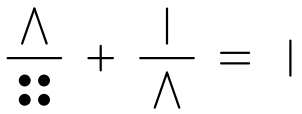
$$\mathbb{Z}/2, \quad C_{2v}, \quad C_{3v}, \quad O(2), \quad O(3).$$

The current fixed-algebra theorem replaces the former claim that every orbit span is irreducible.

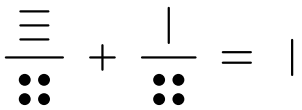
Plate F4 — the right-face bound



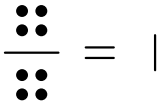
one right face, with a three-quarter-turn defect in the treatise's ledger convention.



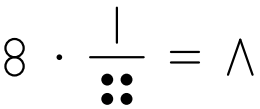
two right faces and a half-turn defect.



three right faces and a quarter-turn defect.

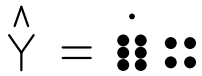


Diagnostic display: four right face angles give zero defect. The current correction does not claim that every flat fold is the unchanged page.

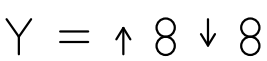


eight quarter-defects total two full turns in the cube-curvature ledger.

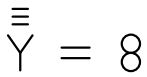
Plate F5 — roots in folded forms



a depressed quadratic with square balancing sixty-four.

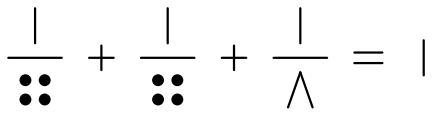


two real roots represented as one magnitude in two vertical orientations.

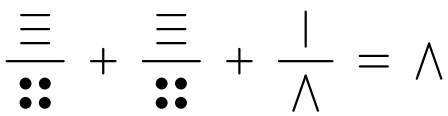


a cubic display with three roots interpreted on a corner carrier.

Plate F6 — the tri-cone partition

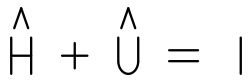


one balance used to display three material identities for a half-plus-two-quarters partition.

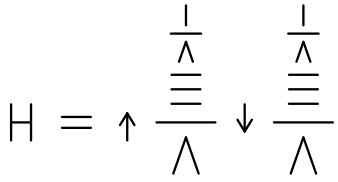


the three displayed deficit terms balance the chosen total-curvature ledger.

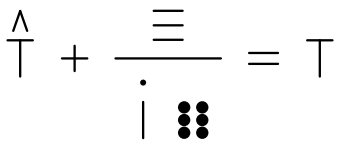
Plate F7 — di-cone variables and heights



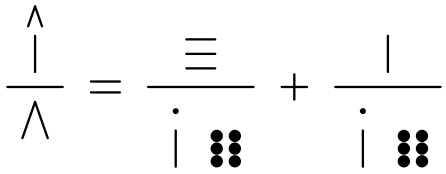
normalized cone relation $h^2 + r^2 = 1$, displayed without a zero-normalized right side.



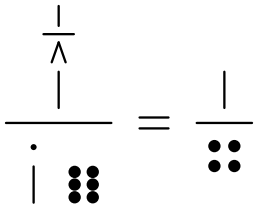
the half-cone height $\sqrt{3}/2$ in two orientations.



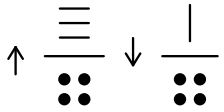
complementary cone fractions with product three-sixteenths.



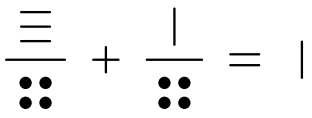
completing the square in the complementary-cone quadratic.



extraction of a quarter-share root.

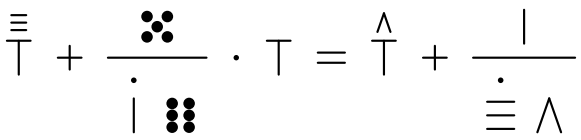


the complementary roots one-quarter and three-quarters as changes around one-half.

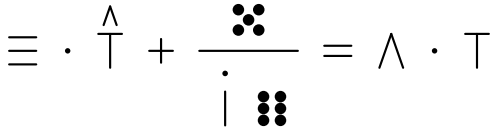


the complementary pair rebalances the source circle.

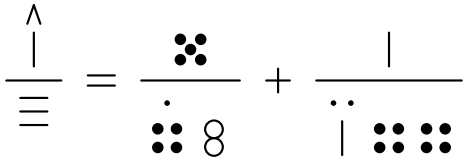
Plate F8 — tri-form cubic



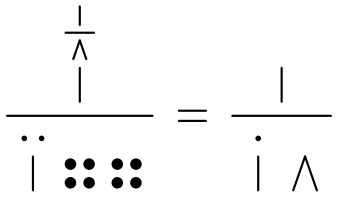
the tri-form cubic in balance form, with roots read as sector fractions.



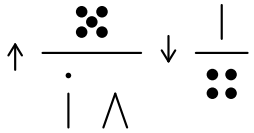
a repeated-root search using the polynomial and its derivative.



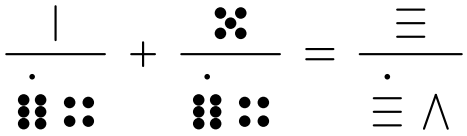
a derivative ladder centered at one-third.



extraction of one-twelfth in the derivative calculation.

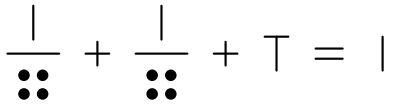


derivative roots at one-quarter and five-twelfths.

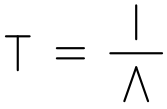


substitution of one-quarter into the two sides of the cubic.

equality of the substituted sides and the double-root interpretation.

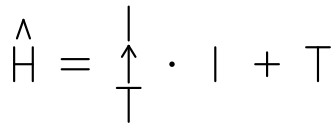


deflation by a repeated quarter-root.



the remaining root one-half.

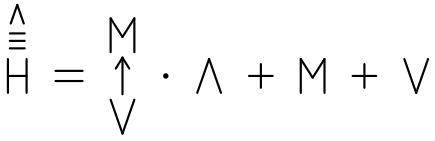
Plate F9 — heights from coefficients and the historical Blank stratification



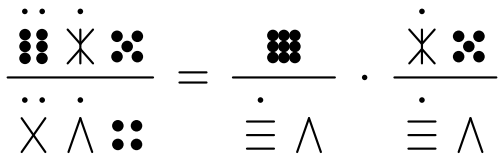
the cone-height identity

$$H^2 = (1 - T)(1 + T),$$

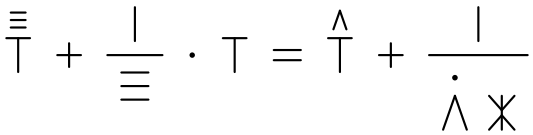
read as a geometric mean of the two classical factors.



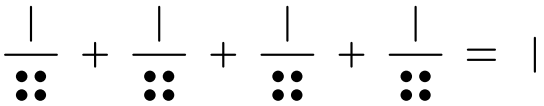
the product of three normalized cone heights expressed through symmetric coefficient data.



a finite tri-cone coefficient identity, reported as six-hundred-seventy-five over one-thousand-twenty-four.



an equilateral tri-form with a triple root at one-third, interpreted as a cusp-type repeated-root stratum.

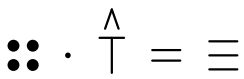


four equal quarter-shares interpreted as a quadruple-root stratum. The catastrophe-theory comparison is interpretive.

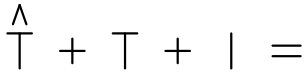
Plate FX — seam law and carrier tower

the sine addition law used for a shared generatrix,

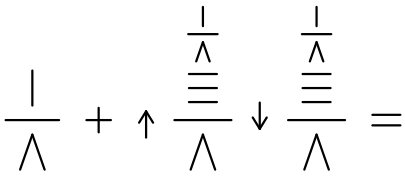
$$\sin(\alpha_1 + \alpha_2) = \sin\alpha_1 \cos\alpha_2 + \sin\alpha_2 \cos\alpha_1 .$$



the equilateral-chain parameter satisfies $4\kappa^2 = 3$.



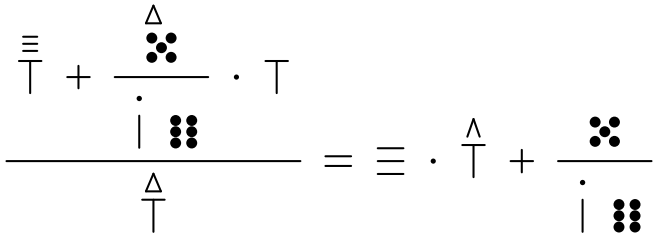
Diagnostic display: a proposed flat-page equation with no positive solution. The withheld side is a renderer diagnostic.



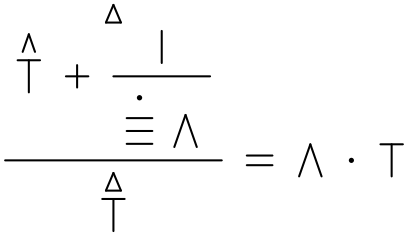
a corner-carrier representation of the two non-real cube roots. The fundamental theorem of algebra remains a classical import.

Plate F11 — differentiation and integration

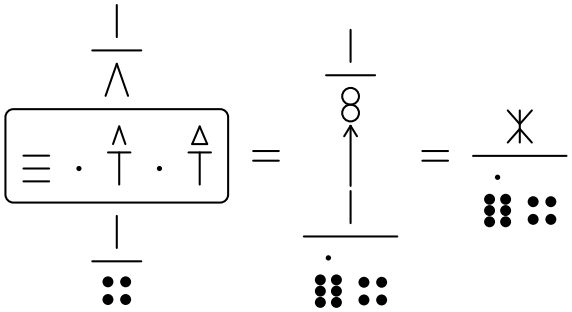
a derivative represented as a fraction of adhered differential marks. Existence still requires the usual analytic limit hypotheses.



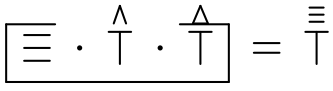
differentiating the displayed cubic-side expression to obtain three squares plus five-sixteenths.



differentiation of the second side to obtain twice the variable.

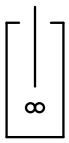


a definite integral over the displayed interval, evaluated as seven-sixty-fourths.

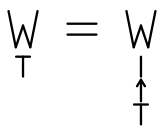


an open enclosure for an indefinite accumulation. The current grammar does not append an additive Blank operand.

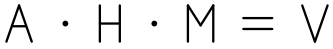
Plate F12 — the mirror axis



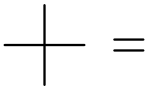
the critical line as a vertical fixed carrier of indeterminate extent.



the functional-equation face swap represented as a balance.

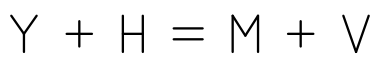


nonvanishing of a finite product in its declared domain. Classical Euler-product zero-freeness still requires its analytic convergence hypotheses.

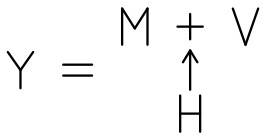


Diagnostic display: the native axis statement rendered as an axis-crossing condition with withheld evaluation. The current strict sentence quantifies over the parameter and completion certificate; the empty side is not part of the sentence.

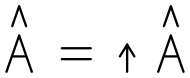
Plate F13 — historical spend-covering display



an explicit-formula balance. The prime and $g(0)\log\pi$ terms retain their classical signs under the current normalization.



universal ascent of this change was asserted in the former proof. It is equivalent to an open positivity condition and is not derived by the current grammar.

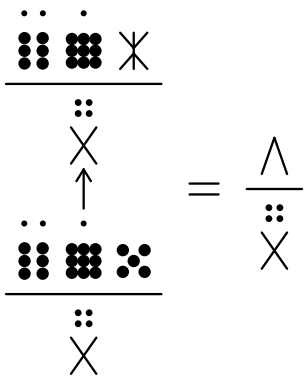


a real on-axis contribution represented as a square.

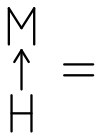
a superposed-orientation phase proposed as an off-axis failure mode. A finite detector family is not complete.

Plate F14 — historical presence-rendered ledger

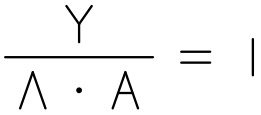
Historical caption corrected by the current record: compact support below $\log 2$ has no contributing prime term; a Gaussian has a small but nonzero prime bill.



a finite-rank comparison between two nearby ledger quantities. It is a reported computation, not an interval certificate.

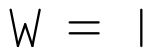


Diagnostic display: agreement at sixteen displayed ranks was drawn as a withheld side. The current numerical record describes finite agreement, not semantic nonemission.



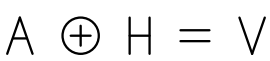
the reported margin divided by twice the first listed spectral weight approaches one in the finite Gaussian table. The global asymptotic remains conditional on tail control.

Plate F15 — historical native derivation I

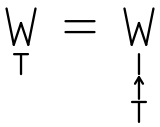


Step one (development record): the descent was normalized to the multiplicative unit in the far-right Euler region.

Step two (development record): the Euler product was displayed as a fusion of prime-labelled factors. The former caption incorrectly invoked the withdrawn Recoverability theorem.



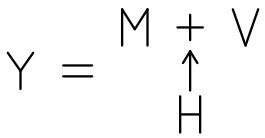
Step three (development record): a no-absorber slogan was used to represent finite-product nonvanishing. Analytic Euler-product nonvanishing still requires convergence and the classical product argument.



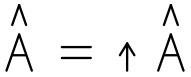
Step four (development record): the functional equation supplied the mirror reading.

Plate F16 — historical native derivation II

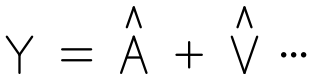
Step five (development record): the explicit formula was identified with a positive-sector junction residue. This was the unproved representation step.



Step six (development record): universal ascent was inferred from the assumed positive representation. This is circular at the classical interface and is not an assertion.



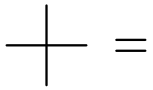
Step seven (development record): page-carried contributions were represented as squares. The inference excluding every off-page contribution was not proved.



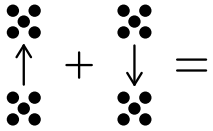
Step eight (development record): the classical spectral interpretation was placed at the end as a "shadow." In the current architecture that interface carries the exact price $I(\mathrm{Adm}_{\zeta}) \Longleftrightarrow \mathsf{RH}_{\mathbb{C}}$.

Plate F17 — historical Witness–Blank statement

Diagnostic display: the finite witness channel had no record in the inspected range. Non-record is not a proof that no counterexample exists.



Diagnostic display, superseded stratum. Nonemission is a metalanguage judgment; the universal claim requires a separate proof.



an annihilation-style diagnostic. Such a diagnostic does not exclude a positioned off-axis exit record.

Plate F18 — historical formal proof I

[historical target]

Step one (development record): the axis sentence was stated as the proof target.

[descent normalization]

Step two (development record): normalization at the multiplicative unit.

[withdrawn Recoverability]

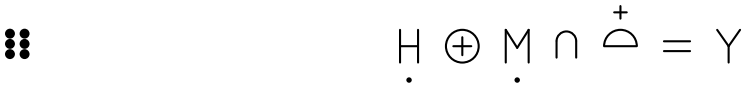
Step three (development record): prime fusion, formerly called lossless by the refuted Recoverability theorem.

[finite nonvanishing]

Step four (development record): finite fused presences were displayed as nonvanishing.

[functional equation]

Step five (development record): the mirror sweep.



[unproved residue representation]

Step six (development record): the junction residue form. This is where the open positivity content entered.

Plate F19 — historical formal proof II

Historical reductio text: the implementation rejected a Blank control as an adhesion host and as an oriented operand. Those rejections do not reject a presented off-axis parameter with a metalinguistic nonemission judgment.

[historical ascent step]

Step seven (development record): ascent was inferred from the assumed positive residue typing.

[invalid symmetry inheritance]

Step ten (development record): the proof inferred that property-free nonemission must be mirror-fixed. The Symmetry No-Go refutes this inference.

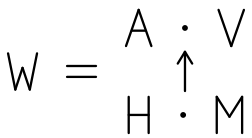
[invalid conclusion]

Step eleven (development record): the witness channel was declared empty. That conclusion does not follow from the preceding grammar checks.

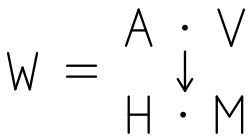
[historical seal]

Step twelve (development record): a sealed empty diagnostic frame was used as a proof tombstone. In the current grammar the frame is a diagnostic preterm and proves no object-language proposition.

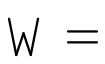
Plate F1X — determinant and turn showcase



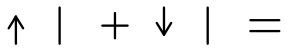
a two-by-two determinant encoded as the oriented change from the odd permutation fusion to the even permutation fusion.



swapping two rows exchanges parity classes and reverses the determinant orientation.



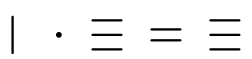
Diagnostic display: equal even and odd permutation fusions. Under a classical ring interface this is the singularity condition.



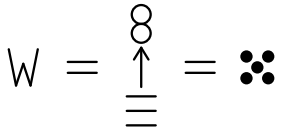
Historical Euler diagnostic: a unit and its half-turned unit annihilate. This represents a boundary of $\mathbb{C}^{\times}$, not the construction of the full field without its additive identity.

Plate F21 — numerical determinant example

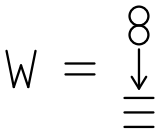
for the array with entries two, one, three, and four, the even product is eight.



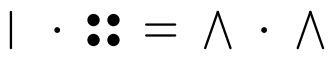
the odd product is three.



the oriented change from three to eight has magnitude five.



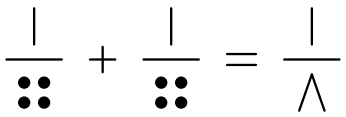
row exchange reverses the determinant orientation.



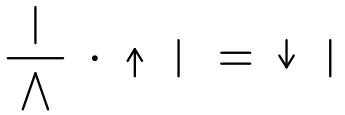
a singular two-by-two example with equal parity products.

Diagnostic display: the corresponding determinant emission is withheld in the parity-pair display.

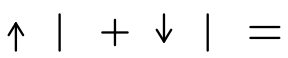
Plate F22 — turn algebra



two quarter-turns compose to a half-turn.

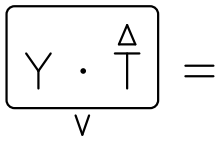


a half-turn reverses the vertical orientation of a unit carrier.

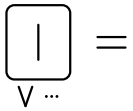


Diagnostic display: opposed unit carriers annihilate.

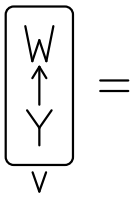
Plate F23 — measure without a native null object



Diagnostic display: an accumulation over a measure-null event. Classical nullity requires the standard measure axioms and hypotheses.

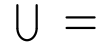


Diagnostic display: countable-union nullity. The result stands on countable subadditivity, not on Blank syntax.



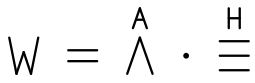
modification of an integrand on a null event. The classical theorem requires measurability and integrability.

Historical Borel–Cantelli premise: a finite sum of event measures.



Historical Borel–Cantelli conclusion: the limsup event is measure-null under the classical theorem's hypotheses.

Plate F24 — historical right-folding tower



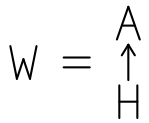
right-foldable group order of the form $2^a 3^b$.

a composition tower with cyclic factors of orders two and three.

Superseded stratum: the generic quintic obstruction is the nonabelian simple factor A_5, not the prime five alone.

Plate F25 — historical determinant proofs

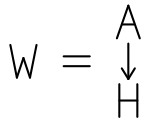
Historical Proof I: alternation as orientation reversal.

		[parity-pair definition]

The determinant encoding is the oriented change from the odd permutation fusion to the even permutation fusion.

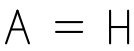
		[parity exchange]

A row transposition exchanges the even and odd permutation classes.

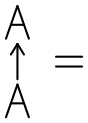
		[orientation reversal]

The parity-pair orientation reverses.

Historical Proof II: singularity as equality of parity fusions.

		[equal parity fusions]

The even and odd permutation fusions are equal.

		[coincident endpoints]

The oriented change from a quantity to itself has no emitted magnitude.

		[diagnostic conclusion]

Under a classical ring interface this equality is equivalent to singularity. The empty diagnostic side is not a strict term.

Plate F26 — historical Euler turn proof

Historical Proof III: the half-turn annihilation display.

[turn addition]

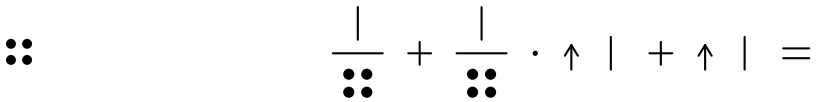
Two quarter-turns compose to a half-turn.

[orientation reversal]

The half-turn carries an ascending unit carrier to its descending orientation.

[annihilation diagnostic]

Opposed unit carriers yield a diagnostic withheld emission.



[assembled diagnostic]

the two quarter-turns and a unit were assembled into an Euler-style annihilation display.

Plate F27 — historical modification theorem

Historical Proof IV: modification on a null event.

[null-event premise]

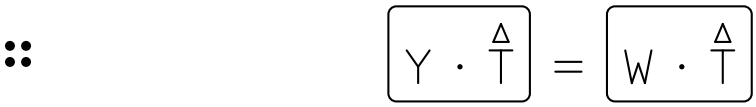
The integrands differ only on an event assumed null under the classical measure interface.

[agreement off the event]

Off the exceptional event the two integrands agree.

[measure import]

The classical conclusion uses additivity and null-set invariance, apparatus of the classical stratum; the grammar generates by emission of formed terms, and the derivation is priced at the interface.



[modification conclusion]

The two classical accumulations agree.

Plate F28 — historical Borel–Cantelli and Burnside proofs

Historical Proof V: Borel–Cantelli in the proposed descent display.

[finite-measure premise]

The series of event measures is assumed finite.

[tail measure]

The classical tail measures tend to the additive identity.

[classical domination]

The limsup event is bounded by the tail measure under the classical Borel–Cantelli proof.

[null limsup]

The classical conclusion is that the limsup event has measure zero.

Historical Proof VI: the Burnside right-folding criterion.

[order $2^a 3^b$]

Assume the group order has only the primes two and three.

[Burnside and composition factors]

Burnside solvability and Jordan–Hölder give cyclic factors of orders two or three.

[converse order product]

Conversely, a tower with only such factors has order $2^a 3^b$.

[A_5 obstruction]

Historical wall display. The current caption identifies the nonabelian simple factor A_5 as the radical obstruction.

Plate F29 — historical imported-lemma discharge

Historical Lemma A: finite-stage genealogy.

[genealogy mark]

Historical claim: every finite inscription carries a stage trace. This does not establish that every classical counterexample relation has the proposed finite address encoding.

Historical Lemma B: Selection Jump display.

[finite fusion]

A finite fusion of stage records remains a finite stage record.

[diagnostic noncompletion]

Historical conclusion: finite fusion does not by itself form an omega-family. This survives only as the rule-relative Selection Jump, not as a proof of the axis statement.

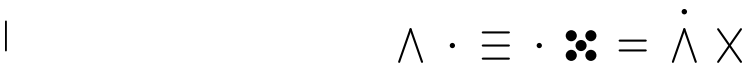
Historical Lemma C: no occupied Blank control.

[implementation WF]

The implementation control does not become a semantic object. The discipline of record excludes any non-emission mark at every layer (WF6).

Plate F2X — Euclid's infinitude of primes

Historical notation execution of Euclid's theorem.



[finite prime product]

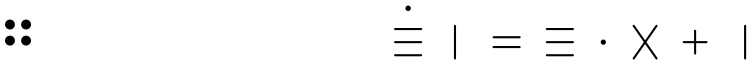
The displayed example multiplies two, three, and five to obtain thirty.

[add the unit]

The finite product is increased by one, producing thirty-one.

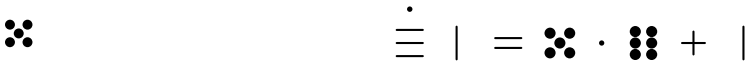
[remainder by two]

Division by two leaves remainder one.



[remainder by three]

Division by three leaves remainder one.



[remainder by five]

Division by five leaves remainder one.

[coprimality step]

A common divisor of the product and the incremented product would divide their difference, the unit.

[new prime divisor]

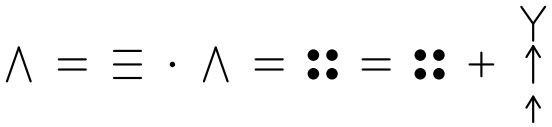
A prime divisor of the incremented product is absent from the chosen finite prime roster. The prime-divisor existence lemma is a classical import.

Chapter 9

The Continued Plates: F31–F56

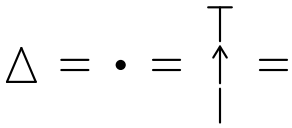
Plate F31 — predicted terms

Historical generated reading: three carrying an upper plus mark balances T, followed by three and an H-carrier. Verdict: open schema; an assertion stands beyond it.



Historical generated reading: a chain of balances ending in an oriented change. Verdict: open schema.

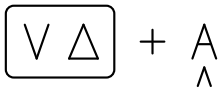
Historical generated reading: six overlaid with a tail-event carrier. Verdict: term; a proposition stands beyond it.



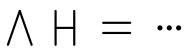
Historical generated reading: a difference mark, rank mark, and oriented-change balance. Verdict: open schema.

Plate F32 — predicted terms

Historical generated reading: H over five. Verdict: term; a proposition stands beyond it.



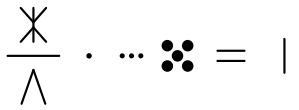
Historical generated reading: an enclosed pair plus a squared variable. Verdict: term; a proposition stands beyond it.



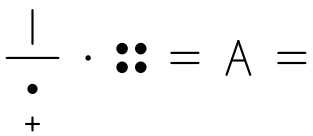
Historical generated reading: two juxtaposed with H, followed by a balance and continuation. Verdict: open schema.

Plate F33 — predicted terms

Historical generated reading: a marked five, two products, and an oriented change. Verdict: term or open schema; a proved identity stands beyond it.

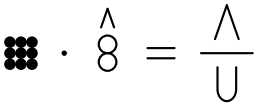


Historical generated reading: seven over two, continuation, five, and one. Verdict: open schema.

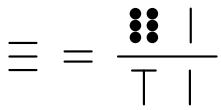


Historical generated reading containing a lower mark, four, and a squared variable. Verdict: nonassertional generated term.

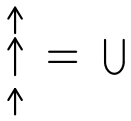
Plate F34 — predicted terms



Historical generated reading: nine times a squared eight, balanced against a fraction involving a tail-event carrier. Verdict: open schema.

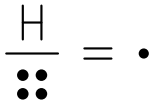


Historical generated reading: three balanced against a compound fraction. Verdict: open schema.

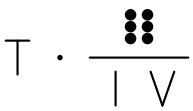


Historical generated reading: an oriented change balanced against a tail-event carrier. Verdict: open schema.

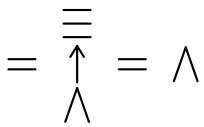
Plate F35 — predicted terms



Historical generated reading: $H/4$ balanced against a rank mark. Verdict: open schema.

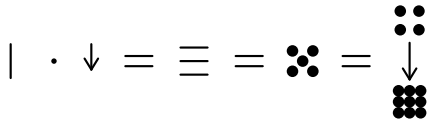


Historical generated reading: T multiplied by a compound fraction. Verdict: term; a proposition stands beyond it.

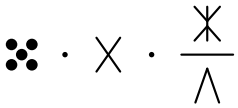


Historical generated diagnostic: a withheld side balanced against an oriented change and a two. It is not a strict sentence.

Plate F36 — predicted terms



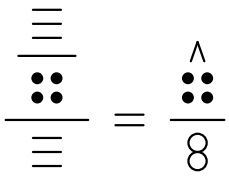
Historical generated reading: one, a descending mark, three, five, and a change from four to nine. Verdict: open schema.



Historical generated reading: five times ten times seven-over-two. Verdict: term; a proposition stands beyond it.

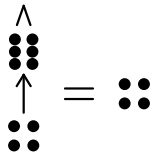
Historical generated reading: seven plus W. Verdict: term; a proposition stands beyond it.

Plate F37 — mined exact identities



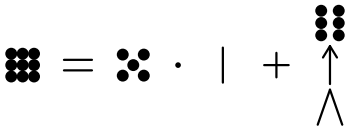
Exact finite identity:

$$\frac{(3/4)}{3} = \left(\frac{4}{8}\right)^2 = \frac{1}{4}.$$



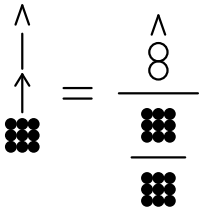
Exact finite identity:

$$(6-4)^2 = 4.$$



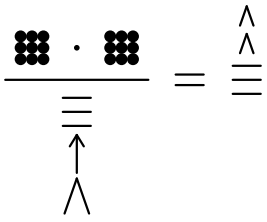
Exact finite identity:

$$9 = 5 + (6-2).$$



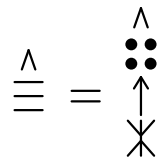
Exact finite identity:

$$(9-1)^2 = \frac{8^2}{9/9} = 64.$$



Exact finite identity:

$$\frac{9 \cdot 9}{3-2} = (3^2)^2 = 81.$$

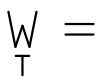


Exact finite identity:

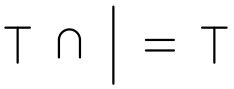
$$3^2 = (7-4)^2 = 9.$$

Plate F38 — exit-locus retyping

Historical authored correction: an exit-locus parameter is a presented, addressable object. In the current strict account it is paired with a completion certificate.

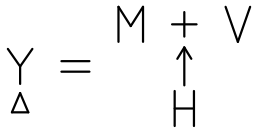


Historical diagnostic: evaluation at the presented parameter has withheld emission. The empty side is metasyntactic.

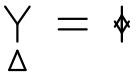


Historical authored axis form: an exit-locus parameter meets the mirror axis in itself.

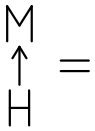
Plate F39 — historical detector



Historical detector margin: the oriented difference between the two explicit-formula sides.



Historical proposed off-axis fingerprint: a superposed turning phase. The detector is partial and finite-range.



Historical finite-range record: no turning was reported over the tested scales. Silence of a finite detector is not a global exclusion certificate.

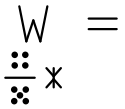
Plate F3X — historical open frontier

Historical candidate Admission display. At the zeta instance its positivity content is equivalent to the classical hypothesis and is adopted only through explicit sponsorship.

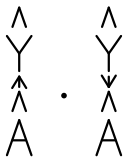
Historical finite witness-channel display.

Historical open diagnostic frame. The frame records an unfinished metasyntactic display and is not a strict object-language proposition.

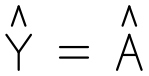
Plate F41 — the Symmetry No-Go canonized



A presented off-axis parameter may carry a completed-evaluation certificate while evaluation withholds a $\mathbb{C}^{\times}$-value. WF6 does not exclude this form.



The polynomial witness is assembled with full mirror symmetry.



The witness has an off-axis root orbit despite the symmetries.

Plate F42 — historical genealogy filters

Historical proposed self-meet classification of the polynomial witness. It excludes one example from one syntactic class but does not prove positivity for the remaining class.

Historical proposed two-genealogy display for the completed zeta descent.

Historical Native Admission candidate. In the current record the zeta-specific instance is explicitly sponsored and its classical price is exactly RH.

Plate F43 — historical adjudication display

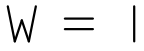
The historical source announced that the external assessment had been adjudicated, the first MJA stratum corrected, Blank discipline tightened, the fixed-algebra theorem adopted, and machine claims rescoped. The claim of completed analytic budgets does not hold at the stratum of record, and Option B Blank is superseded by the strict renderer-control classification.

Historical witness-channel display: finite non-record through a reported range, not a universal exclusion certificate.

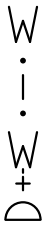
Historical prime-genealogy ladder. The mirror constrains; arithmetic and analysis must supply any zeta-specific axis theorem not adopted as an axiom.

Plate F44 — historical exclusion filters

Historical first filter: the polynomial symmetry witness was classified as a self-meet. This typing excludes that witness from a chosen class but does not prove positivity on the class.



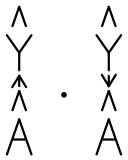
Historical second filter: a Davenport–Heilbronn-type function was classified as carrying a genealogy other than the required prime-product one.



Historical candidate Admission domain: distinct genealogy, prime-fused trace, and positive-sector conditioning. Sufficiency is the adopted or open clause, not a consequence of the filters.

Plate F45 — historical discharge specification

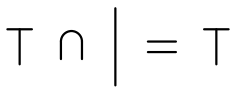
Historical specification: a successful analytic discharge must couple the exit-locus functional to the prime genealogy rather than use mirror symmetry alone.



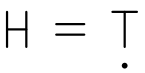
The symmetry-only witness remains available and blocks every mirror-only proof.

Historical finite witness-channel record.

Plate F46 — historical Keeping theorem



Historical clause one: the native axis sentence.

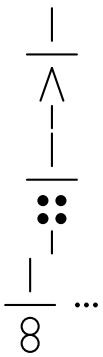


Historical clause two: a proposed finite negative witness. The existence of such a witness depends on the exact effective criterion used.

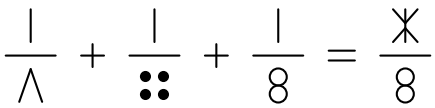
Historical clause three: selected finite affirmation strategies were closed relative to the historical corpus. This did not exhaust every possible proof method.

Historical clause four: finite non-record. The current positive status chapter separately states the conditional arithmetic independence route and its exact domain.

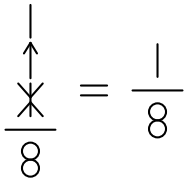
Plate F47 — Zeno stages and seal



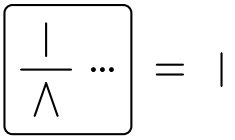
Step one (development record): a geometric ladder of finite stages.



Step two (development record): a finite geometric partial-sum identity.



Step three (development record): the finite geometric remainder equals the next stage.



Step four (development record): the completed geometric sum was enclosed. For the omega witness channel, an analogous completed family requires an explicit completion law rather than this finite identity alone.

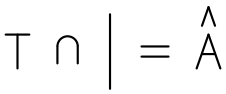
Step five (development record): proposed stage rejection and finite counterexample branches.

Step six (development record): a sealed diagnostic frame.

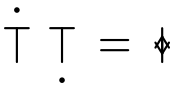
Step seven (development record): closed and open diagnostic frames displayed side by side. The current omega register prices closure through the explicit sponsorship postulate.

Plate F48 — historical orbifold positivity examination

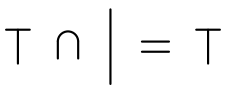
Step one (development record): a self-convolution test interpreted as a mirror fusion.



Step two (development record): an on-axis real spectral contribution represented as a square.



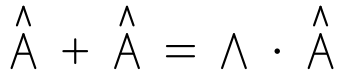
Step three (development record): an off-axis pair represented on two sheets with a turning phase.



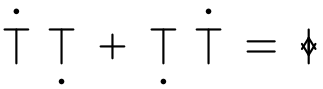
Step four (development record): the critical strip interpreted as a reflection orbifold, with the critical line as seam.

Step five (development record): seam contributions versus off-seam sheet pairs. Excluding every sheet pair is exactly the open or adopted Admission content.

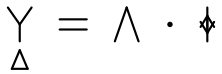
Plate F49 — historical conservation examination



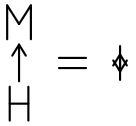
Step one (development record): an on-axis conjugate pair represented as a two-cone orbit with doubled square weight.



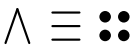
Step two (development record): a complete off-axis quartet has a real combined contribution under a symmetric test.



Step three (development record): the quartet contribution was described as real, amplified by horizontal displacement, and oscillatory in the test scale.

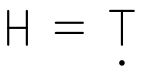


Step four (development record): the treatise argued that a specified quartet would eventually affect a suitable real test margin. This does not make the finite detector complete.

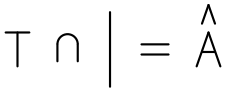


Step five (development record): a four-sheet/right-folding comparison. The analytic content is carried by the quartet formula, not by the folding analogy.

Plate F4X — historical compressed native argument



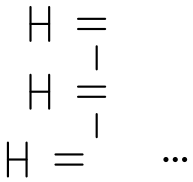
Step one (development record): a proposed constructive falsity witness. Its scope depends on the exact effective criterion and test construction.



Step two (development record): on-axis square contribution.

Step three (development record): implementation rejection of an occupied Blank control. It does not decide the location of exit parameters.

Step four (development record): the entire remaining argument was compressed to one Admission cell. That cell is precisely the sponsored native law and has classical price RH.



Step five (development record): an omega-length ledger was proposed as a completed proof object. Finite prefixes do not form the completed family without the registered completion event.

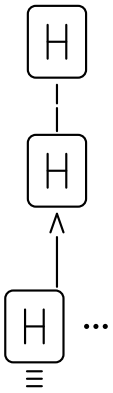
Plate F51 — historical sponsored occupation

Historical sponsored seal. The current interpretation is exact: the provenance mark records adoption of a completion or native law; it does not represent an analytic derivation from weaker premises.

Historical display of a sponsored native verdict beside an open classical analytic channel. Each register stands in its own authority.

Plate F52 — finite stages and omega-family

A finite kept-stage record with its level mark.



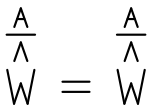
A proposed coherent omega-family. The current witnessed semantics forms it only through the explicit completion sponsor.

Historical side-by-side analytic and omega silhouettes. Their coextensiveness requires the named clash-channel and analytic interfaces.

Plate F53 — historical reference clauses

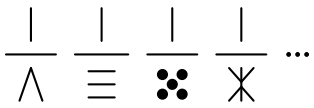
Historical genealogy clause: one local factor record per prime.

Historical harmonic or pole-normalization clause.

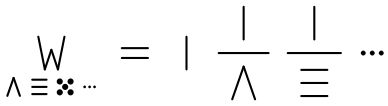


Historical mirror-completion clause. Reference follows only under the analytic hypotheses of the named Hamburger converse theorem.

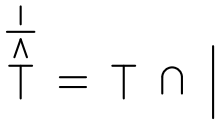
Plate F54 — historical architecture in five displays



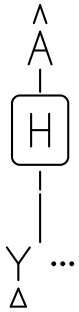
Historical first display: prime-indexed cone ratios.



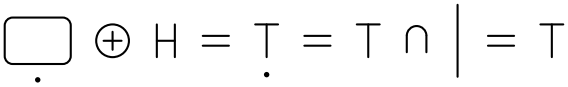
Historical second display: prime genealogy and harmonic normalization.



Historical third display: the half-share tent and the critical seam.

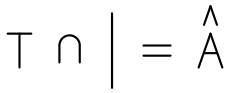


Historical fourth display: several proposed coextensive silhouettes. Each equivalence requires its own interface theorem.

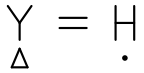


Historical fifth display: sponsorship, finite witness typing, and the native axis statement in one term. The current theorem derives the axis sentence from the sponsored Adm_{ζ}; the classical transfer remains exactly priced.

Plate F55 — class-wide independence models



Model A: roots on the seam give positive field contributions at every presented point right of the seam.



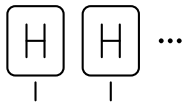
Model B: the exact witness

$$\Phi\left(\frac{3}{5}+5i\right)=-\frac{7821885}{3131252}<0.$$

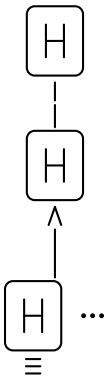
The class-wide native fork. This theorem does not by itself prove zeta-specific or external arithmetic independence.

Historical open enclosure above the two class-wide models.

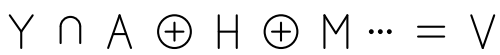
Plate F56 — historical transport to an omega-family



Historical fibre records over finite addresses.



Historical atlas/family comparison. The formal Transport theorem is conditional on an explicit dictionary.



Historical shared-clause display. A corpus-specific identification requires inspection of the external record definitions; it is not proved by the abstract transport theorem alone.

Chapter 10

The Caption-Blind Decoding Audit

This chapter is a documentary record of a caption-blind trial of the object language, together with its exact evidential status. The trial is part of the volume's provenance: its one conviction is the recorded origin of the stricter Blank control now in force, so the current stratum's discipline cites this chapter as the register's first external audit instrument.

10.1 The trial

The glyph corpus — the figures, stripped of every caption, label, and word — was delivered to an independent reader in a fresh context, with staged probes designed to leak neither vocabulary nor target. The reader was caption-blind, not mathematics-blind: the correct condition, since the reference clauses of Section 24.3 concern informed interpreters. Three rounds; findings at exact strength.

Round one: arithmetic. The bijective numerals, the rank-dot place system, the carry law, commutativity, and the machine edition's quadratic ladder were recovered, and every operator was correctly typed. The language's arithmetic stratum self-decodes.

Round one's conviction. Offered a vacant slot in an additive frame — three plate emissions then carried a blank fused as an addend — the reader introduced a null object, performing exactly the category collapse the typing charge indicts. The layout, not the reader, was convicted: the emissions were repaired, and the stricter Blank control of the current stratum is the enactment. The experiment thereby served as the register's first external audit instrument.

Round two: the primes. The membership rule of the index family was recovered exactly — membership above one admitting only the unit factorization, with the square-root stopping rule derived from the reflected pairing of divisors: the mirror law's arithmetic shadow, surfacing unprompted. The reader generated correct unattested members from the decoded rule: the grammar is generative in foreign hands.

Round two: the retraction. Tested distributionally, the null-object hypothesis was formally retracted: the empty enclosure is never added, multiplied, exponentiated, indexed, or oriented; it stands only as a balance side or within an enclosure. The reader concluded that it marks a well-typed place whose content is absent — the blank-as-judgment doctrine, reached from evidence alone by a classically trained mind, with the distinction between an unfinished syntax and an inscribed absence drawn without prompting. The sponsored seals were read as meta-level provenance marks, as designed.

Round three: the mirror and the naming. The reader corrected its own earlier census, reformulated the reflection as an involutive order-reversing operation under which the notation is closed —

legality preserved, statements carried to equivalent or dual statements — and identified this closure as the structural law, with pointwise symmetry its fixed-point case: the mirror theorem, recovered blind at language level. Then, holding the prime index, the reciprocal ladders, the privileged half with its fixed-point property, and the reflection law jointly, the reader named the object: the Riemann zeta function and its Euler product, at a self-reported confidence of ninety percent; the completed descent with its reflection about one half, seventy; an approach to the critical line and the hypothesis, sixty. The figures the reader specified as settling the remainder are, item for item, the categoricity ingredients of Section 24.3 and the axis statement of the Riemann architecture: the reader requested exactly the plates the volume contains.

10.2 What the trial establishes, and what it does not

The exhibition claim — that the notation carries arithmetic, generative grammar, and structural law to an independent mind without one word of any human language — gains empirical support. The reference clauses gain their trial: the categoricity ingredients, jointly held, moved a blind reader to the intended object by name. And the register gained an instrument: the one violation surviving fourteen internal audits was found by external eyes in one round.

Status, recorded exactly. The trial had sample size one; the reader was a single language model; confidence figures are self-reported; the protocol was not a blinded human-subject experiment. The record is an exploratory audit observation and supports the exhibition and reference claims at evidence grade; it is not a theorem of semantic legibility or of reference, and no clause of this volume's assertion of record rests on it. Within those limits the finding stands as stated: reference travels in the strokes; the keeping, as ever, is the cell.

Part V

Typed Algebra and Geometry

Chapter 11

The First Junction Model and Its Failure

The junction algebra was first proposed with a surcharge cost law, distribution of intersection over fusion, a residue endomorphism, and a Recoverability theorem. This chapter states that model exactly and refutes it; the typed successor is defined in the next chapter, and every later use is governed by the successor.

11.0.1 Carriers

Definition 11.1 (Sorts)**.** MJA has three sorts:

- Sw: *sweeps*, the objects written $\langle \partial u \times v_\infty \rangle$ — an oriented pairing of a differential element ∂u with an infinity carrier v_∞ (a ray of indeterminate extent, Plate 9's unterminated stroke).
- Res: *residues*, finite inscriptions; the presence-fusion $\oplus : \mathsf{Res} \times \mathsf{Res} \to \mathsf{Res}$ is total, commutative, associative (*stipulated*), with *no* neutral element: $(\mathsf{Res}, \oplus)$ is a commutative semigroup, not a monoid — the Against-Zero signature.
- $\mathcal{K}$: *costs*, a commutative ordered semigroup $(\mathcal{K}, \odot, \preceq)$ of positive magnitudes with no neutral element; "no cost" is not a cost element but the non-inscription of a cost mark.

Non-inscription ε is not an element of any sort; it is the meta-level name for the absence of a term. Operations below are *partial*, and "the result is ε" abbreviates "the expression has no value and no element is written."

11.0.2 Axioms

Axiom 11.2 (Commutativity — forced by the grammar)**.** $X \cap Y = Y \cap X$ *whenever either side is defined. This is not optional:* $\cap$ *is an achiral atom occupying a* $\mathsf{Row}(;)$ *slot, and WF3 licenses it only for commutative denotations. The strict notation* constrains *the algebra before the algebra is written.*

Axiom 11.3 (Typing of the junction)**.** $\cap$ *is defined on: (i)* $\mathsf{Sw} \times \mathsf{Sw} \to \mathsf{Res}$ — *the junction of two sweeps precipitates a residue; (ii)* $\mathsf{Res} \times \{ \pi^+ \} \to \mathsf{Res}$ — *conditioning by the positive sector (Axiom 11.4); and is undefined (hence* ε*) on disjoint-support pairs (Axiom 11.7(c)).*

Axiom 11.4 (Positive-sector projection)**.** π^+ *(the drawn upper sector carrying a glued +) acts on residues idempotently,* $\pi^+\pi^+ = \pi^+$*, commutes with* $\oplus$*,* $\pi^+(R_1 \oplus R_2) = \pi^+R_1 \oplus \pi^+R_2$*, and every residue precipitated by a junction is written already conditioned:* $\mathsf{Sw} \cap \mathsf{Sw}$ *lands in the image of* π^+*. This is why the corpus form always closes with* $\cap S_r^+$*: the sector is the presence-guarantee, the projection that certifies the residue as an inscribable magnitude. (*Stipulated*, matching the corpus shape.)*

Axiom 11.5 (Distribution)**.** *Sweeps are additive in the differential slot,* $\langle \partial(u{+}u') \times v_\infty \rangle = \langle \partial u \times v_\infty \rangle \oplus \langle \partial u' \times v_\infty \rangle$*, and the junction distributes over fusion where defined:* $X \cap (R_1 \oplus R_2) = (X \cap R_1) \oplus (X \cap R_2)$*. Thus* $(\oplus, \cap)$ *form a* presence hemiring*: semiring axioms minus both identities. (*Stipulated.*)*

Axiom 11.6 (Cost valuation)**.** *Each defined junction carries a cost* $\kappa(X \cap Y) \in \mathcal{K}$ *obeying*

$$\kappa(X \cap Y) \;=\; \kappa(X) \odot \kappa(Y) \odot \delta(X, Y),$$

where the defect $\delta(X, Y)$ *is either non-inscribed (no mark; the transverse case) or an element of* $\mathcal{K}$ *(the tangential surcharge). Associativity of* $\cap$ *holds* up to cost bookkeeping*:* $(X \cap Y) \cap Z$ *and* $X \cap (Y \cap Z)$ *precipitate the same residue, with equal total cost, whenever all junctions involved are defined. (*Stipulated*; verified in the model, Theorem 11.12.)*

Axiom 11.7 (Transversality trichotomy)**.** *For sweeps* $S_1 = \langle \partial u \times v_\infty \rangle$*,* $S_2 = \langle \partial w \times x_\infty \rangle$ *exactly one holds:*

(a) Transverse*: the elements and carriers are jointly independent. Then* $S_1 \cap S_2$ *is defined,* δ *is non-inscribed, and the residue is the fused pair of traces conditioned by the sector:* $S_1 \cap S_2 = (A \oplus B) \cap \pi^+$ *with* $A = \mathrm{tr}(S_1)$*,* $B = \mathrm{tr}(S_2)$*.*

(b) Tangential*: the supports overlap without coinciding. The junction is defined,* $\delta(S_1, S_2) \in \mathcal{K}$ *is inscribed, and the residue carries a merged trace.*

(c) Disjoint*: the supports do not meet. The junction is undefined; the expression is* ε *— blank paper, not a zero object.*

The self-junction $S \cap S$ *is the tangential extreme: it precipitates only* $\mathrm{tr}(S)$ *and its cost is pure surcharge.*

Axiom 11.8 (Genealogy)**.** *A trace morphism* $\mathrm{tr} : \mathsf{Res} \to \mathcal{T}$ *into the descent-trace semigroup satisfies* $\mathrm{tr}(R_1 \oplus R_2) = \mathrm{tr}(R_1) \cdot \mathrm{tr}(R_2)$ *and records, for every residue, the constraint path by which it descended from the plenum* Ω_Λ*. Residues are path-remembering; junctions append constraints to the genealogy. (*Stipulated*, transcribing the descent foundation.)*

11.0.3 The recoverability theorem

First-Model Claim 11.9 (First junction model: Recoverability claim)**.** *Let* $S_1 \cap S_2$ *be defined. The pair* (S_1, S_2) *is recoverable from the data* $\big(S_1 \cap S_2,\ \kappa(S_1),\ \kappa(S_2)\big)$ *if and only if* $\delta(S_1, S_2)$ *is non-inscribed.*

Proof. ($\Leftarrow$) In the transverse case the residue is $(A \oplus B) \cap \pi^+$ with A, B the separate traces (Axiom 11.7a); by Axiom 11.8 the genealogy of a fused pair is the product of genealogies, and in the transverse case the two factors have independent constraint paths, so the factorization of $\mathrm{tr}(A \oplus B)$ into its two coprime paths is unique and each factor, together with its cost, determines its sweep.

(⇒) In the tangential case the merged trace of Axiom 11.7b identifies the overlapping constraint once; the surcharge δ is exactly the inscription of what the merge forgot, and distinct pairs sharing the same overlap and the same complements precipitate identical residues, so no inverse exists. Formally, in the blade model of §11.0.4 the tangential fiber of the junction map over a fixed residue has positive dimension equal to dim of the overlap, while the transverse fiber is a single point; the model computation is Theorem 11.12(iv). □

Remark 11.10 (Settling the arity dispute). The earlier objection stands confirmed: syntactic arity is vacuous — $\cap$ is always binary on the page. The operative kept is δ: a junction is a *genuine* (lossless) meet exactly when its defect is non-inscribed, and "lossy merge" now has a definition, a measure, and a theorem, rather than a gesture.

11.0.4 The blade model and soundness

Definition 11.11 (Blade model)**.** Fix an oriented real inner-product space E of finite dimension with a distinguished open convex cone C (the positive sector). Interpret:

- an infinity carrier v_∞ as the ray $\mathbb{R}_{>0}v$ (a point of the positive projective sphere — a "line of indeterminate length": direction without scale, endpoint never inscribed);
- a sweep $\langle \partial u \times v_\infty \rangle$ as the decomposable 2-blade $u \wedge v$ with its orientation, i.e. the oriented plane spanned by the element and the carrier;
- $\oplus$ as oriented direct sum of blades on independent supports and formal fusion otherwise;
- $S_1 \cap S_2$ as the oriented intersection of the two planes, conditioned by C: the residue is the pair of unit traces of the planes on the intersection, pushed into C (π^+ = intersect with the cone);
- κ = codimension counted in the zero-free semigroup $(\mathbb{Z}_{\geq 1}, +)$ (a free junction of two planes in general position in E has the minimal cost, one unit per constrained dimension); δ = the dimension of excess overlap beyond general position, non-inscribed when that excess is empty.

First-Model Claim 11.12 (First junction model: blade-model soundness claim)**.** *In the blade model: (i) Axioms 11.2–11.8 hold; (ii) $\cap$ is commutative and associative-up-to-cost as stipulated; (iii) the trichotomy of Axiom 11.7 is the standard trichotomy of pairwise position of planes (transverse, partially overlapping, disjoint-in-the-cone), with (c) yielding an empty cone-intersection — no element to inscribe; (iv) the recoverability biconditional of Theorem 11.9 holds, the tangential fiber over a residue having dimension equal to the overlap excess. Hence* MJA *is consistent: it has a model.*

Proof. (i)–(iii) are elementary linear geometry: intersection of subspaces is commutative; codimensions add under transverse intersection and exceed additivity by the overlap dimension otherwise, which is exactly Axiom 11.6 with δ = excess; the cone conditioning is an idempotent operation commuting with direct sum on independent supports, giving Axiom 11.4. (iv): in the transverse case two planes in general position through a common line are determined by their traces on that line together with their codimension data, so the junction map is injective on transverse pairs with fixed costs; in the tangential case one may rotate each plane within the overlap without changing either the intersection or the costs, producing the positive-dimensional fiber. □

11.0.5 The Zeno instance computed

Proposition 11.13 (The corpus form — derived from the axioms)**.** *In the form* $\{\langle \partial\theta \times \vec{r}_\infty \rangle \cap \langle \partial\vec{x} \times \theta_\infty \rangle\} \to \{(A_r \oplus B_r) \cap S_r^+\}$*, the two sweeps are transverse: the angular element is independent of the linear element, and the radial carrier of the first is independent of the angular carrier of the second. Therefore, by Axiom 11.7(a): the junction is defined; the defect is non-inscribed; the residue is exactly the fused pair of traces conditioned by the positive sector — the right-hand side of the corpus form, now* derived *rather than stipulated; and by Theorem 11.9 the junction is lossless: every descended stage retains recoverable knowledge of both sweeps.*

Corollary 11.14 (Zeno)**.** *The course of indeterminate length is notated by its carrier ray (endpoint non-inscribed, Definition 11.11); stages are transverse junctions precipitating lossless residues; the genealogy morphism (Axiom 11.8) orders them by descent from* Ω_Λ*, and every element of* $(\mathsf{Res}, \oplus)$ *and of* $(\mathcal{K}, \odot)$ *is a presented residue or a presented cost: these are semigroups, and each carries its own least presented member. The supertask is set aside rather than performed: completion is exhaustion of constraint, and what exhaustion presents is a completion certificate — a positive record that the declared procedure ran to its end.*

Chapter 12

The Typed Algebra MJA_2

12.1 From the failed model to the typed algebra

The surcharge law, the distributivity axiom, and Recoverability fail in the blade model; the counterexamples of the preceding chapter close that route. The algebra of record is the typed system defined here.

The assertion is the smaller typed algebra MJA_2.

12.2 Signature of MJA_2

Definition 12.1 (Typed carriers)**.** Fix the following metalanguage sorts.

- Sw, the sort of proper sweeps.
- $\mathsf{Res} \sqsubseteq \mathsf{Sw}$, the subsort of proper nontrivial residues.
- $(\mathcal{K}, \oplus_{\mathcal{K}})$, a positive commutative semigroup of costs.
- $(\mathcal{T}, \sqcup)$, a trace monoid.
- Obs, a sort of positively presented observations.

The partial operations are:

$$\oplus : \mathsf{Sw} \times \mathsf{Sw} \rightharpoonup \mathsf{Sw},$$

$$\cap : \mathsf{Sw} \times \mathsf{Sw} \rightharpoonup \mathsf{Res},$$

$$\kappa : \mathsf{Sw} \to \mathcal{K}, \qquad \mathrm{tr} : \mathsf{Sw} \to \mathcal{T},$$

and

$$c_+ : \mathsf{Res} \rightharpoonup \mathsf{Obs}.$$

Fusion is defined only when the fused sweep remains proper. Junction is defined only when the intersection residue is proper and nontrivial. Undefinedness is a metalanguage fact about the partial operation; no null residue is returned.

12.3 Axioms

Axiom 12.2 (Partial commutative fusion)**.** *Where defined, $\oplus$ is commutative and associative.*

Axiom 12.3 (Modular cost)**.** *Whenever all displayed terms are defined,*

$$\kappa(X \cap Y) \oplus_{\mathcal{K}} \kappa(X \oplus Y) = \kappa(X) \oplus_{\mathcal{K}} \kappa(Y).$$

Axiom 12.4 (Genealogy)**.** *Where fusion is defined,*

$$\mathrm{tr}(X \oplus Y) = \mathrm{tr}(X) \sqcup \mathrm{tr}(Y).$$

Axiom 12.5 (No proper absorber)**.** *No member of Sw is an absorber for every defined fusion.*

Distribution of $\cap$ over $\oplus$, Recoverability of inputs from a residue, and conditioning as a residue endomorphism are not axioms of MJA_2.

12.4 Conditioning

Let E be an ambient classical carrier and $C_+ \subset E$ a fixed positive cone. In the metalanguage define

$$P_+(X) = X \cap C_+.$$

The operator P_+ is monotone and idempotent on its set-valued domain:

$$P_+(P_+(X)) = P_+(X).$$

The observation sort contains the presented nonempty cone sections of residues. The map

$$c_+(R) = P_+(R)$$

is a partial corestriction, defined only when the section is observation-typed. The classical metalanguage may state that another section is empty; the strict object language receives no empty observation.

Idempotence belongs to P_+. The expression $c_+ \circ c_+$ is not used when its codomain and domain do not match.

12.5 Finite-dimensional model

Theorem 12.6 (Proper-subspace model)**.** *Let E be a finite-dimensional vector space. Interpret sweeps as proper nontrivial subspaces equipped with presented generating data. Interpret fusion as partial subspace sum, junction as partial intersection, cost as codimension, and trace under fusion as concatenation of generating records. Exclude the trivial subspace and the ambient space from the native carriers. Then the commutative-fusion, modular-cost, genealogy, and no-proper-absorber axioms hold wherever the partial operations are defined.*

Proof. Commutativity and associativity follow from subspace sum on the declared domain. The dimension formula

$$\dim(A + B) + \dim(A \cap B) = \dim A + \dim B$$

is equivalent to

$$\mathrm{codim}(A \cap B) + \mathrm{codim}(A + B) = \mathrm{codim}\, A + \mathrm{codim}\, B,$$

which gives the modular cost law whenever both partial outputs belong to their native carriers. Genealogy holds by the definition of the presented generator record under fusion. The absorber for subspace sum would be the ambient space, which is excluded as improper. □

Remark 12.7. The model theorem is a theorem of ordinary finite-dimensional linear algebra. It does not interpret the Riemann explicit formula, prove Weil positivity, establish Recoverability, or make MJA_2 a general representation of arithmetic.

Chapter 13

The Reading Group: Folding the Page

13.1 Fixed-function formulation

Theorem 13.1 (Invariant-algebra reading theorem)**.** *Let a group G act on a reading surface Σ. Scalar content that descends to the quotient is a function on Σ/G, equivalently a G-fixed function on Σ. For the polynomial actions used in this volume,*

$$\mathbb{R}[x,y]^{\mathbb{Z}/2} = \mathbb{R}[x^2, y],$$

$$\mathbb{R}[x,y]^{C_{3v}} = \mathbb{R}[r^2, \mathrm{Re}(z^3)],$$

and

$$\mathbb{R}[x,y]^{O(2)} = \mathbb{R}[r^2].$$

Proof. For page reflection $x \mapsto -x$, an invariant polynomial contains only even powers of x, giving $\mathbb{R}[x^2, y]$. The real reflection group C_{3v}, acting as the dihedral group of order six, has polynomial invariants generated by $r^2 = x^2 + y^2$ and $\mathrm{Re}(z^3) = x^3 - 3xy^2$. The $O(2)$-invariant polynomials are radial and therefore polynomials in r^2. These are standard fixed-ring computations; see [72]. □

An orbit span decomposes into isotypic components and may decompose further; irreducibility stands apart: irreducible. In particular, the span of a two-element chiral orbit under $\mathbb{Z}/2$ decomposes as trivial plus sign. The older claim that every orbit span is itself an irreducible chirality is superseded.

The tent, corner, and cone are geometric realizations of the relevant actions. They are not asserted to be literal models of every quotient space carrying the same group.

13.2 The page, tent, corner, and cone

The page carries the reflection group $\mathbb{Z}/2$. A tent adds a second commuting reflection and realizes C_{2v}. A three-face corner realizes C_{3v}. A rotational cone carries an $O(2)$ reading action.

These geometric carriers are interpretations of the strict syntax. The structural Mirror theorem itself is the recursive statement of Chapter 5; it does not depend on physically folding paper.

13.3 Right-face defect

Proposition 13.2 (Right-face convexity bound)**.** *At a strictly convex polyhedral vertex, the sum of incident face angles is strictly less than 2π. If every incident face angle is $\pi/2$, at most three such faces occur.*

Proof. If n right face angles meet, strict convexity requires

$$n\frac{\pi}{2} < 2\pi.$$

Hence $n < 4$, so $n \leq 3$. □

At four right angles the angular defect is zero, so no strictly convex positive-curvature vertex forms. This does not imply that every defect-free fold is physically identical to an unfolded page; nontrivial flat-fold configurations may remain. Any dimensional interpretation of the bound is therefore an interpretation, not part of the convexity theorem.

Chapter 14

Page, Tent, Corner, and Cone

14.1 The reading correspondence, exact form (definitive)

Lawful scalar carriers on a reading surface Σ with reading group G are the G-fixed functions — equivalently, functions on Σ/G — and the fixed algebra (classically, the fixed algebra — the import named once here) is computed per action: for the page reflection, $\mathbb{R}[x, y]^{\mathbb{Z}/2} = \mathbb{R}[x^2, y]$ (height *and* squared offset); for the planar C_{3v} action, generators r^2 and $\mathrm{Re}(z^3)$; for $O(2)$, $\mathbb{R}[r^2]$. The linear span of a G-orbit of marks decomposes isotypically and is in general reducible: the chiral pair's span is trivial $\oplus$ sign. The narrative realization that follows (tent, corner, cone) is a family of geometric models of these actions, and its claims are read through this section.

14.1.1 From one mirror to a group

Parts A–B fixed the reading symmetry as a single reflection: $G = \mathbb{Z}/2$ acting on the plane of the page, with Theorem 5.14 the statement that semantics descends to the quotient. Folding the sheet changes G. Join two sheets at a ridge (the *tent*): the isometry group is $C_{2v} \cong \mathbb{Z}/2 \times \mathbb{Z}/2$ (face-swap and end-swap), and the face-swap realizes the reflection functor ρ as a literal isometry of a three-dimensional object: a term t and its reflection ρt inscribed on the two faces produce a tent that *closes* — faces coinciding under the fold — precisely when t is mirror-coherent. Join three sheets at a corner: $G = C_{3v}$. Roll the sheet and glue its vertical edges: the cone, with $G = C_{\infty v} = O(2)$, rotations about the axis together with reflections through every axial plane. Horizontal position, which on the page carried no kept meaning, becomes angle; a Row wrapped around the cone becomes a cyclic word, and WF3's commutativity discipline strengthens to cyclic keeping.

14.1.2 The classification theorem

Theorem 14.1 (Carriers and chiralities — derived)**.** *Let the notation live on a surface Σ with reading group G acting by isometries, and let semantics be required to descend to Σ/G. Then:*

(i) The lawful carriers of asymmetric content are exactly the G-fixed coordinate functions on Σ (heuristic listing; the exact fixed algebras appear in the definitive section — for the page reflection the full algebra is $\mathbb{R}[x^2, y]$, so squared offset is equally lawful): height on the page; distance from the ridge-axis on the tent; distance from the apex on the corner and the cone; radius on a fully $O(3)$-symmetric inscription space.

(ii) Content legible at every reading position spans the trivial isotypic component of the glyph space under G.

(iii) The possible kinds of chirality are classified by the nontrivial irreducible representations of G. For the page ($G = \mathbb{Z}/2$) there is exactly one: the sign representation — identifying orientation *as the unique irreducible chirality of the flat system, which is why circulation repeatedly surfaced as the one datum the flat notation could not absorb. For C_{3v} and $O(2)$ new classes appear: the rotation-sense representation and the two-dimensional doublets (E, resp. E_m per angular wavenumber), pair-classes with no flat counterpart.*

(iv) The 1D chiral system discarded in favor of the strict grammar is recovered as representation theory: achiral glyphs spanned the trivial isotype, chiral pairs spanned the sign isotype. Folding does not remove chirality; it refines its classification.

Proof sketch. (i) A coordinate can carry meaning readable from every G-position iff it is constant on G-orbits of reading frames, i.e. G-kept; for the listed groups the fixed algebras are generated by the stated distance functions. (ii) Descent of semantics to Σ/G is keeping of denotation under G, which on the linear span of glyph-content is projection to the trivial isotype. (iii)–(iv) Decompose the glyph space into irreducibles; a G-orbit of marks denoting converse-related content is exactly a copy of a nontrivial irreducible, and for $\mathbb{Z}/2$ the only one is the sign. □

Corollary 14.2 (Against Zero, geometrized — derived)**.** *Every folded form has a locus fixed by all of G: ridge axis, corner point, apex, center. At that locus every reading frame coincides, and coincidence is what the locus exhibits. An inscription forms where a frame is distinguished; the surface the notation reads is therefore the surface of distinguished frames, in which the fixed locus is present as the place of coincidence. The arithmetic counterpart is the tally: a tally forms over a presented roster, and counting accordingly begins at one. Both are formation rules with a positive least case, and the parallel is between those two rules. The fixed locus is a point and the tally is a count; the geometry and the arithmetic share a shape of formation rather than an object. The affine line's want of a distinguished basepoint belongs to a third register again: a choice withheld, which is a gauge, and gauge is settled by adoption.*

Proposition 14.3 (The bound: at most three right cones — derived)**.** *A corner folded from flat material exists iff its face angles at the vertex sum to strictly less than a full turn (positive angular deficit; the convex vertex condition, Descartes). A right cone-face consumes a quarter turn. Hence n right faces demand $n \cdot \frac{\pi}{2} < 2\pi$, i.e. $n \leq 3$: one right face (deficit three quarters), two (the tent's ridge end, deficit one half), three (the cube corner, deficit one quarter) — and at $n = 4$ the deficit is* non-inscribed*: the quarters exhaust the circle and folding returns the flat page. The fourth right cone fails by blankness, not by contradiction. Since three mutually perpendicular faces at a point are three perpendicular directions, the third dimension is characterized as the last inscribable right-cone deficit, and the folded family of right corners in Theorem 14.1 terminates: page, tent, cube corner. Descartes' closure completes it: eight quarter-deficits balance two full turns ($8 \cdot \frac{\pi}{2} = 4\pi$), the eight corners of the cube exhausting the sphere's total curvature.*

Remark 14.4. The termination is itself a presence-only statement: the flat limit at $n = 4$ is the non-event of folding — deficit balancing blank, page returned — exactly parallel to the annihilation, closed-loop, and coincident-bound exit loci of the flat calculus. Dimensionality, on this reading, is bounded by what deficit remains inscribable.

Remark 14.5 (Orbifold form). Tent, corner, and cone are the quotients of the plane by $\mathbb{Z}/2$, C_{3v}, and a rotation group; a notation on the folded form is a notation on the quotient orbifold, and

the "symmetry search" is the computation of the deck group. Theorem 5.14 is the case $G = \mathbb{Z}/2$; each folding replays it for larger G. Machine realization: the folded plates render each face as a primitive-level reflection or rotation of a single engine emission, so face-consistency is inherited from the verified mirror checks rather than re-drawn.

Chapter 15

Roots, Resolvents, and Folding Towers

Theorem 15.1 (Roots and reading groups)**.** (i) Degree two (derived). *Depress the balance quadratic by the torsor recentering $y = x$ shifted to the parabola's axis (no origin required: the shift is a change, not a coordinate). The depressed equation reads y^2 balances D, and its two roots are one magnitude in two orientations: the ascending and descending readings of a single stack. Galois conjugation is arrow reversal — the sign representation of the page's reading group $\mathbb{Z}/2$ (Theorem 14.1(iii)), now acting on roots.*

(ii) Degree three (derived, classical). *The generic cubic has Galois group $S_3 \cong C_{3v}$, the reading group of the cube corner: the three roots inhabit the three faces, the corner's rotation carries the conjugate pair (the E-doublet), its reflections swap them, and the square root of the discriminant spans the rotation-sense class A_2 — even permutations are the rotations, transpositions the face-reflections.*

(iii) Degree four (derived, classical). *The generic quartic has Galois group S_4, the rotation group of the Descartes-closed cube of Proposition 14.3, acting faithfully on the four space diagonals: the roots* are *the diagonals. The resolvent cubic's three roots are the three pairings of the diagonals — the cube's three axes — the induced action of the resolvent factors through $S_4/V_4 \cong S_3$; the root-stabilizer descent $S_4 \supset S_3$ is the separate one-root picture, not the resolvent construction. standing the cube on a vertex, returning from solid to corner.*

(iv) Torsion accounting (derived). *The composition factors of S_2, S_3, S_4 are exclusively $\mathbb{Z}/2$ and $\mathbb{Z}/3$ — precisely the torsion realized by the right-folded family, whose termination at three faces is Proposition 14.3. This is why the quartic formula requires only square and cube roots. S_5 introduces the factor A_5: not cyclic, indeed simple, and isomorphic to the rotation group of the icosahedron — a form built of fifths, outside right material.*

(v) Correspondence (interpretive, so marked). *The reading that "unsolvability of the quintic and the termination of right folding are the same wall" is a structural correspondence, not a proved equivalence: what is theorem is (i)–(iv); what is proposal is the identification of radical-tower existence with folding-tower existence in general. The correspondence is exact on the classified cases.* Subsequently resolved: *the right-folding case is closed by Burnside's $p^a q^b$ theorem — right-foldability holds iff the Galois group's order is $2^a 3^b$, with existence automatic — see Part XII Resolved, §6, and the formal proof on Plate F28.*

Remark 15.2 (Roots of the identity). The group elements themselves are roots of the variable "symmetry": the page mirror solves $x^2 = \mathrm{id}$, the corner rotation solves $x^3 = \mathrm{id}$, and the characters of the reading groups take values in square and cube roots of unity. The fourth root — the quarter-

turn i — is exactly the element whose right-folded carrier exit loci out at $n = 4$ (Proposition 14.3): the imaginary unit lives at the flatness boundary of right material. Its four quarters exhaust the circle; $i^4 = 1$ is the deficit ledger's blank line read as an equation.

15.1 Quadratic and cubic readings

For a real depressed quadratic

$$y^2 = D$$

with D nonnegative, the two real roots may be read as one magnitude with two vertical orientations. If D lies below zero [the sign here names a presented bound], the roots require the rotational complex carrier; the real page reading does not apply.

The generic cubic has Galois group $S_3 \cong C_{3v}$. The corner therefore supplies a geometric realization of its permutation action. This is a classical group-theoretic fact together with a geometric interpretation; it is not a derivation of the cubic formula from the shape of a corner.

15.2 Quartic resolvent

Proposition 15.3 (Resolvent pairing)**.** *For a generic quartic with roots r_1, r_2, r_3, r_4, the three resolvent quantities correspond to the three partitions of the four roots into two unordered pairs. The induced permutation action factors through*

$$S_4/V_4 \cong S_3.$$

The separate subgroup picture $S_4 \supset S_3$ describes the stabilizer of one root. It is not the construction of the resolvent cubic.

15.3 Vieta and discriminants

The classical fixed-ring theorem gives

$$k[r_1, \ldots, r_n]^{S_n} = k[e_1, \ldots, e_n],$$

where $e_1, \ldots, e_n$ are the elementary symmetric functions [55]. Coefficients are therefore invariant functions of the roots. The present program reads solving as passage from the fixed coefficient data to a genealogy carrying individual roots. That reading is an interpretation of the classical theorem, not an additional representation theorem.

The discriminant detects collision of roots. At a repeated root the classical discriminant equals zero; under a presence interpretation, the positively presented data are the parameter and the collision certificate, while the failed distinction is a metalinguistic nonseparation judgment. No Blank term is substituted for the discriminant value inside the strict language.

15.4 Right-folding

Definition 15.4 (Right-folding tower)**.** A right-folding tower for a finite group G is a composition series whose composition factors are cyclic of order 2 or 3.

Theorem 15.5 (Right-folding order criterion)**.** *A finite group has a right-folding tower if and only if its order is of the form*

$$2^a 3^b$$

for nonnegative metalanguage integers a, b.

Proof. If the group has such a composition series, the product of the orders of its factors is $2^a 3^b$. Conversely, Burnside's $p^\alpha q^\beta$ theorem implies that every finite group of order $2^a 3^b$ is solvable [12]. Every composition factor of a finite solvable group is cyclic of prime order. Since only the primes 2 and 3 divide the group order, all factors are C_2 or C_3. □

15.5 The generic quintic

The generic quintic has Galois group S_5, whose composition series contains the nonabelian simple factor A_5. That factor is the decisive obstruction to solvability by radicals [18]. The obstruction is not the prime five by itself: cyclic C_5-extensions are solvable by radicals.

Right-folding and radical solvability are therefore compared only as an explicitly defined structural analogy. Theorem 15.5 classifies the right-folding family; classical Galois theory supplies the distinct A_5 obstruction. No general equivalence between physical folding constructions and all radical towers is asserted.

Chapter 16

Counting by Positive Analytic Data

This chapter records classical realizations used by the program. Each result remains classical and transfers only through its stated interface.

16.1 Meromorphic counting kernel

Proposition 16.1 (Residue count with exact hypotheses)**.** *Let $\Omega \subset \mathbb{C}$ be a bounded region with positively oriented piecewise smooth boundary. Let $\mathcal{N}$ be meromorphic on a neighborhood of $\overline{\Omega}$, with no pole on $\partial\Omega$. Suppose each enclosed pole represents a presented item and its residue equals that item's multiplicity. Then*

$$\frac{1}{2\pi i}\oint_{\partial\Omega} \mathcal{N}(z)\,dz$$

equals the sum of those multiplicities.

Proof. This is the residue theorem under the displayed hypotheses [3]. The presence interpretation concerns the roster of poles and their multiplicity records; it does not alter the classical theorem. □

The contour-free-of-poles hypothesis and the residue-equals- multiplicity hypothesis are both essential. A merely meromorphic function is not automatically an admissible counting kernel.

16.2 Cyclic factorization and partial fractions

Let $\zeta_m = e^{2\pi i/m}$. The degree-m factorization is

$$n^m - l^m = \prod_{j=1}^{m}(n - \zeta_m^j l).$$

For presented l and away from the poles, the corresponding partial fraction identity in the n-variable is

$$\frac{1}{n^m - l^m} = \frac{1}{m\,l^{m-1}} \sum_{j=1}^{m} \frac{\zeta_m^j}{n - \zeta_m^j l}.$$

Proof. At the pole $n = \zeta_m^j l$, the derivative of $n^m - l^m$ is

$$m(\zeta_m^j l)^{m-1} = m\zeta_m^{-j} l^{m-1}.$$

The residue is therefore $\zeta_m^j/(ml^{m-1})$, which gives the displayed decomposition. □

If one sheet is presented and labelled n_1, the deck orbit is generated genealogically:

$$n_{j+1} = \zeta_m \, n_j \quad (j = 1, \ldots, m-1), \qquad \zeta_m \, n_m = n_1.$$

This orbit supplies a local labelled roster only after a sheet label has been presented.

16.3 Digamma comb

The digamma function $\psi(w) = \Gamma'(w)/\Gamma(w)$ has simple poles at $w = 0, -1, -2, \ldots$, each with residue -1. Consequently

$$\psi(\rho - z)$$

has poles at

$$z = \rho, \rho + 1, \rho + 2, \ldots$$

with positive unit residues in the z-variable.

Indeed, near $z_0 = \rho + k$,

$$\rho - z = -k - (z - z_0),$$

and hence

$$\psi(\rho - z) \sim -\frac{1}{\rho - z + k} = \frac{1}{z - z_0}.$$

The sign depends on the variable. Statements that assign negative unit residues to this comb in the z-variable are superseded.

16.4 Monodromy and labelled rosters

A covering map may have globally constant fiber cardinality while admitting no global labelled trivialization. Monodromy obstructs the latter, not the former.

For example,

$$z \longmapsto z^2 : S^1 \to S^1$$

has fiber cardinality two over every point. The classical degree is globally defined. Traversing the base circle exchanges the two local labels, so there is no global continuous enumeration preserving the chosen sheet identities.

The program calls a roster retaining witness identity a *genealogical roster*. Nontrivial monodromy may prevent such a global labelled roster even though the unlabeled cardinality remains constant. Two lawful responses are available: forget the labels and retain the degree, or pull back to a cover on which a trivialization is presented. Neither response licenses the claim that monodromy destroys global fiber cardinality [38].

16.5 Planar Dirichlet heat trace

Proposition 16.2 (Heat-trace constant)**.** *Let Ω be a smooth bounded planar Euclidean domain with Dirichlet boundary condition. As $t \downarrow 0$,*

$$\operatorname{Tr}(e^{t\Delta_D}) \sim \frac{|\Omega|}{4\pi t} - \frac{|\partial\Omega|}{8\sqrt{\pi t}} + \frac{\chi(\Omega)}{6} + \cdots .$$

In particular, the constant term is $\chi(\Omega)/6$.

This is a classical heat-kernel result under the stated smooth, bounded, planar, Euclidean, and Dirichlet hypotheses [44, 56, 30, 79]. Other dimensions, boundary conditions, corners, singular metrics, or non-Euclidean geometries require their own formulas.

Chapter 17

The Dagger Identity: the Root Algebra Returned

The referent of the program's deep logic connections is recorded here at adjudicated strength. The provenance verdict stands: the cross-domain schema is interpretive. This chapter therefore asserts its two internal columns as theorems of the present volume, imports its third column as classical mathematics named at the point of use, and declares the identification of pattern as a named reading with its instances tabulated exactly — not as a theorem.

17.1 The schema

Definition 17.1 (Dagger reading)**.** In each of three columns, semantics factors through an involution-generated reading group, and the content expressible without an orientation choice is the fixed component of the action:

geometry	G-keeping	everywhere-legibility on the folded form
analysis	mirror-coherence	reflection-kept semantics
arithmetic	Galois-keeping	base-field definability

The *dagger reading* is the declared identification of this shared factorization pattern across the columns.

Status by column. The geometric column is Theorem 13.1: lawful carriers on a reading surface are the fixed functions of the reading group, computed per action. The analytic column is the reflection metatheory of the strict grammar: involutivity (Theorem 5.14), preservation of well-formedness (Theorem 5.15), and renderer equivariance (Theorem 5.16). The arithmetic column is classical Galois theory, imported: the fixed field of the Galois group is the base field, and the fixed ring of the symmetric action on roots is the ring of coefficients. The identification of the three factorizations as one schema is the reading declared above; it is exact on every instance tabulated in this volume and is asserted at no greater strength.

17.2 Vieta re-read

The classical fixed-ring theorem of Chapter 15,

$$k[r_1, \ldots, r_n]^{S_n} = k[e_1, \ldots, e_n],$$

acquires its reading here. Coefficients are the mirror-coherent content of roots: a balance-form equation, which inscribes coefficients, is an achiral inscription, and solving it is a controlled descent into chirality. The moment an orientation glyph appears is the moment the notation reaches past the fixed component; a single root is never inscribable without exhibited orientation, only the achiral pair or an oriented member carried by Up/Dn. Solving is breaking mirror-coherence in a licensed way. Status: this is the fixed-ring theorem re-read as a statement about the language; the mathematics is classical, and the reading is the dagger reading applied to one instance.

17.3 The boundary of the reading

The reading is a factorization schema, not a solvability equivalence. Theorem 15.5 classifies the right-folding family, and classical Galois theory supplies the distinct A_5 obstruction for the generic quintic; no general equivalence between physical folding constructions and radical solvability is asserted, exactly as the roots chapter records. The dagger reading survives that boundary because it never crossed it: what is identified is the pattern of keeping, not any transfer of solvability between columns.

Part VI

Cone-Chain Applications

Chapter 18

Closed Planar Cone Chains

Conventions as in the di-cone papers: a sector of fraction tinteriortotheunitinterval of a circle of radius R rolls into a right circular cone of slant height R, base radius $r = Rt$, semi-vertical angle α with $\sin\alpha = \kappa = t$. "Material" of a cone family is $m = \sum_k \sin\alpha_k$, in units of one circle's arc. A planar chain requires consecutive shared generatrices, $\gamma_k = \alpha_k + \alpha_{k+1}$; a closed chain requires $\sum_k \gamma_k = 2\pi$, i.e. $\sum_k \alpha_k = \pi$.

18.1 The no-go and the two-circle barrier

Proposition 18.1 (Single-circle no-go)**.** *Cones cut from a single circle satisfy $\sum_k \sin\alpha_k = 1$. Since* arcsin *is convex on $[0,1]$ with $\arcsin 0 = 0$, it is superadditive, hence*

$$\sum_k \alpha_k = \sum_k \arcsin(t_k) \;\le\; \arcsin\Big(\sum_k t_k\Big) = \arcsin(1) = \frac{\pi}{2} \;<\; \pi.$$

No closed planar chain exists from one circle; the angle chain of any single-circle family spans at most a quarter turn, with equality only in the degenerate limit of a single flat cone.

Proposition 18.2 (Two-circle infimum; non-attainment)**.** *Over all closed planar chains (any $n \ge 3$, $\alpha_k \in (0, \pi/2)$),*

$$\inf m \;=\; \inf\Big\{ \textstyle\sum_k \sin\alpha_k : \sum_k \alpha_k = \pi \Big\} \;=\; 2,$$

and the infimum is not attained. Proof. sin *is strictly concave on $[0,\pi/2]$, so on the polytope $\{\alpha \in [0,\pi/2]^n : \sum \alpha_k = \pi\}$ the concave functional m attains its minimum at a vertex; every vertex has two coordinates equal to $\pi/2$ and the rest 0, giving $m = 2$. Both boundary values are degenerate (flat disk, needle), so within nondegenerate cones the value 2 is approached, never reached.* □

Remark 18.3. Thus closure costs strictly more than two circles of arc, and there is *no* unconstrained minimizer: cheap closures degenerate. A minimizer exists only after a regularity selection. Two natural selections follow; the first is geometric symmetry, the second is quantum admissibility.

18.2 The minimal symmetric closure

Theorem 18.4 (Equilateral tri-chain)**.** *Among closed planar chains of n congruent cones, the material is $m(n) = n\sin(\pi/n)$, strictly increasing in $n \ge 3$. Hence the minimal symmetric closure is unique: $n = 3$,*

$$\alpha = \frac{\pi}{3}, \qquad \kappa = \frac{\sqrt{3}}{2}, \qquad m_{\min} = \frac{3\sqrt{3}}{2} \approx 2.598,$$

three congruent $60°$ *cones, sector angle* $\pi\sqrt{3} \approx 311.7°$ *each, axes coplanar at mutual angle* $\gamma = 120°$ *through the common apex. The configuration's full symmetry group is* D_{3h}*, containing the reading group* C_{3v} *of the corner.* Proof. *Congruence forces* $\alpha_k = \pi/n$*;* $m(n) = n\sin(\pi/n)$ *increases to* π*;* $n = 3$ *is least admissible since a* 2*-chain closes only at the doubly degenerate* $\alpha_1 = \alpha_2 = \pi/2$. □

Proposition 18.5 (Closure forces a saddle apex)**.** *At the common apex of any closed chain the gathered intrinsic angle is* $\sum_k 2\pi\kappa_k = 2\pi m > 4\pi$ *by Proposition 18.2: the apex carries angle* excess *(concentrated negative curvature). Closure and positive-deficit (single-circle) material are incompatible — the no-go of Proposition 18.1 restated intrinsically: planar closure is bought at the price of a hyperbolic vertex.*

Remark 18.6 (The offcut cone)*.* Cutting the three $\pi\sqrt{3}$-sectors from three circles leaves three congruent remainders totalling fraction $\kappa_{\text{off}} = 3 - \frac{3\sqrt{3}}{2} \approx 0.402$ of one circle — which itself rolls into a well-formed right cone ($\alpha_{\text{off}} \approx 23.7°$). No material is wasted: the minimal closure plus its offcut cone is an exact partition of three circles.

18.3 Quantum structure of the closed tri-chain

Theorem 18.7 (Inherited self-adjoint family, conditional)**.** REGISTER REASSIGNED. *Assume the global operator domain is the compatible assembly of the three local seam domains and the modewise apex domains, with the time-reversal and current-conservation restrictions stated below. Then: Equip each sheet* C_κ*,* $\kappa = \sqrt{3}/2$*, with the da Costa Hamiltonian* H_κ *of the di-cone paper. The closed tri-chain admits the interface family*

$$\mathcal{F} \;=\; U(2)^{\,3} \times \mathcal{E}_{\text{apex}},$$

one $U(2)$ *per seam via the unitary boundary condition* $(I - U_k)\Phi_k + iL(I + U_k)\Phi'_k = 0$*, and* $\mathcal{E}_{\text{apex}}$ *the modewise apex-extension family in the limit-circle channels. Every member yields a self-adjoint Hamiltonian with conserved seam currents.* Proof. *The seam theorem of the di-cone paper is local: its hypotheses (locality, linearity, time-reversal keeping, current conservation) are verified seam-by-seam, and the three seams are disjoint away from the apex; the apex is handled by the standard limit-point/limit-circle classification with* $\nu_n = \kappa^{-1}\sqrt{n^2 + (1-\kappa^2)/4}$*. At* $\kappa = \sqrt{3}/2$*:* $\nu_{\text{low}} = \frac{\sqrt{3}}{6} \approx .289$*, interior to the unit interval, so the lowest channel of every sheet is limit-circle and* $\mathcal{E}_{\text{apex}}$ *is a nontrivial* $U(3)$*-type boundary family on the three-dimensional lowest-channel apex space; channels* $|n| \geq 1$ *have* $\nu_n \geq \nu_1 = \sqrt{51}/6 \approx 1.19 > 1$*, limit-point, no data needed.* □

Remark 18.8 (Threshold roots)*.* The rank of the apex family jumps exactly at the algebraic thresholds $\nu_n = 1$, i.e. $\kappa^2 = (4n^2+1)/5$: the square roots $\kappa = \sqrt{(4n^2+1)/5}$ are the "roots of the variable" at which the quantum boundary algebra changes dimension. Only $n = 0$ threshold $\kappa = 1/\sqrt{5}$ lies in the open unit interval; sheets sharper than $\arcsin(1/\sqrt{5}) \approx 26.57°$ have *no* apex freedom at all. The minimizer's $60°$ sheets sit safely above it.

Theorem 18.9 (Bloch sectors labelled by cube roots of unity)**.** *Restrict* $\mathcal{F}$ *to the* C_3*-equivariant subfamily (equal seam couplings* $U_k \equiv U$*, apex condition commuting with the cyclic rotation* $\mathcal{R}$ *of sheets). Then* $[H, \mathcal{R}] = 0$ *and the Hilbert space splits into eigenspaces of* $\mathcal{R}$ *with eigenvalues* $1,\ \omega,\ \omega^2$ $(\omega = e^{2\pi i/3})$*; the dynamics and each modewise S-matrix block-diagonalize,*

$$S_n \;=\; S_n^{(1)} \oplus S_n^{(\omega)} \oplus S_n^{(\omega^2)},$$

three eigenphase families labelled by the cube roots of unity. The closed chain thus supports a genuinely new observable absent from open chains: chain holonomy*, the Bloch phase accumulated*

around the three seams. Proof. *Standard spectral decomposition of a unitary $\mathbb{Z}/3$ symmetry commuting with a self-adjoint H; block structure of S follows from equal-carrying of the boundary condition.* □

Proposition 18.10 (Spin-holonomy parity; the tri-cone is anomaly-free)**.** *For any single-circle partition into n cones, the deficits satisfy $\sum_k \delta_k = 2\pi(n-1)$, so the product of apex spin holonomies is*

$$\prod_k e^{i\delta_k/2} = e^{i\pi(n-1)} = (-1)^{n-1}.$$

*$n = 2$ recovers the di-cone's global -1 (the anomaly the seam-twisting hypothesis is designed to cancel); $n = 3$ gives $+1$: **the tri-cone — half the circle into two cones, half into one — is the minimal single-circle partition whose total spin holonomy is trivial.** The anomaly-cancellation machinery becomes unnecessary precisely at the first odd partition.*

Proposition 18.11 (The anomaly is the offcut)**.** *For the minimal symmetric closure built from three circles, the total spin holonomy is*

$$\prod_{k=1}^{3} e^{i\delta_k/2} = e^{\,i\pi\,(3-\sum_k \kappa_k)} = e^{\,i\pi\,\kappa_{\text{off}}}, \qquad \kappa_{\text{off}} = 3 - \tfrac{3\sqrt{3}}{2},$$

i.e. the residual spin phase of the closed chain equals π times the offcut fraction: the anomaly of the minimal closure is exactly the inscription of its unused material. Proof. *Immediate from $\delta_k = 2\pi(1-\kappa_k)$ and the definition of κ_{off}.* □

18.4 The root algebra of the di-cone, and the deep connections

Proposition 18.12 (The di-cone is a quadratic)**.** *The complementary parameters $\kappa_1 = t$, $\kappa_2 = 1-t$ are the two roots of*

$$x^2 - x + p = 0, \qquad p = t(1-t),$$

with discriminant $\Delta = 1 - 4p = (2t-1)^2$. The sheet-swap $\psi_1 \leftrightarrow \psi_2$ is the Galois conjugation of this quadratic; the heights $h_i = R\sqrt{1-\kappa_i^2}$ are square roots whose sign choices are the up/down orientations of the two nappes (the sign representation). The balanced split $\theta = \pi$ is exactly the branch point $\Delta = 0$ *(double root $\kappa = \frac{1}{2}$), and the di-cone paper's balanced-split phenomena are the degeneration of the Galois action there: the conjugate pair collapses, and the surviving structure is the irrep decomposition of the swap — the even/odd sectors into which the seam-coupled problem decouples, and in which $M_n(k)$ becomes scalar and S_n diagonalizes.* Status: derived (elementary verification against the quantum di-cone theorems).

Remark 18.13 (Junction dictionary). The apex/seam coupling $\Lambda = UBU^*$ realizes, inside standard quantum mechanics, the meet-junction algebra of the notation program: the extension/coupling parameter is the junction's inscribed spend; Dirichlet decoupling $U = -I$ is the disjoint case (no junction; balances blank); off-diagonal U (tunneling) is the lossless transverse junction, channel data recoverable; and the rank jumps at the threshold roots $\kappa = \sqrt{(4n^2+1)/5}$ are the points where the junction's boundary algebra itself changes. *Status: structural correspondence, exact on the stated items; not a formal equivalence.*

Remark 18.14 (The deep-logic ledger). Collected connections, each anchored above: (i) degree two: sheet-swap = Galois of the κ-quadratic; balanced split = branch point (Prop. 18.12); (ii) degree

three: the minimal symmetric closure is threefold (Thm. 18.4), its reading group contains $C_{3v} \cong S_3$, and its quantum sectors are labelled by the cube roots of unity (Thm. 18.9) — the same roots that permute the cubic's conjugates on the corner's faces; (iii) parity: single-circle partitions carry spin holonomy $(-1)^{n-1}$, making the tri-cone the first anomaly-free partition (Prop. 18.10); (iv) presence accounting: closure is forbidden to positive-deficit material (Prop. 18.5), the unconstrained minimum does not exist (Prop. 18.2), the symmetric minimizer wastes no element (offcut cone), and its residual anomaly *is* the offcut (Prop. 18.11): the phase remembers the unused material.

Part VII

Riemann Architecture

Chapter 19

Formation of the Native Riemann Question

19.1 Classical analytic object

In the classical register define

$$\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma\Big(\frac{s}{2}\Big)\,\zeta(s).$$

It is entire and satisfies

$$\xi(s) = \xi(1-s)$$

and

$$\xi(\bar{s}) = \overline{\xi(s)}.$$

The standard analytic background is given in [66, 75, 27].

Let

$$\Xi(t) = \xi\left(\frac{1}{2} + it\right).$$

For real t, $\Xi(t)$ is real.

19.2 Presence retyping

Definition 19.1 (Presence interpretation of evaluation)**.** The presence interpretation does not interpret ξ as a total map into a field containing a null value. It retypes the classical evaluation as a partial map

$$\xi_{\mathrm{P}} : \mathsf{Par}_{\mathrm{pres}} \rightharpoonup \mathbb{C}^{\times}.$$

At a presented parameter s, if the classical interface supplies a value of $\mathbb{C}^{\times}$, evaluation emits that value:

$$\mathrm{Eval}(\xi_{\mathrm{P}}, s) \downarrow \xi(s) \in \mathbb{C}^{\times}.$$

If the completed evaluation procedure finishes at s but emits no member of $\mathbb{C}^{\times}$, the metalanguage records

$$\mathrm{Eval}(\xi_{\mathrm{P}}, s) \uparrow .$$

The symbol $\uparrow$ is a judgment about the partial evaluation. It is not an element of $\mathbb{C}^{\times}$, a third truth value, or an object-language term.

Definition 19.2 (Exit-locus record)**.** An exit-locus record is a pair

$$\mathsf{Exit}_{\xi}(s; c)$$

consisting of:

1. a positively presented parameter s;
2. a certificate c that the declared completed evaluation procedure ran to completion at s.

The associated nonemission is retained in the metalanguage as $\mathrm{Eval}(\xi_{\mathrm{P}}, s) \uparrow$.

The certificate is positive data. The parameter is positive data. The withheld value stays a metalanguage judgment.

19.3 Nonformation of the literal zero-locus sentence

Proposition 19.3 (No direct additive identity in the tally sort)**.** *Let $T \cong \mathbb{N}_{>0}$ be the positive tally semigroup with*

$$m \oplus n = m + n.$$

Then every element of T moves every element of T.

Proof. For all $m, n \in T$,

$$m \oplus n = m + n > m.$$

Thus no $n \in T$ satisfies $m \oplus n = m$ for every m. □

Proposition 19.4 (Literal nonformation)**.** *The strict grammar G_{P} has every term formed by an act, with a numerical zero term and a sentence constructor comparing a presence-valued evaluation with such a term. Therefore*

$$\mathsf{RH}_{\mathrm{C}}^{(0)} \notin \mathrm{Sent}(\mathsf{G}_{\mathrm{P}}).$$

Proof. Inspect the atom registry and the twelve constructors of Definition 5.8. No zero atom occurs, and no constructor introduces one. The result follows by closure of the term grammar. □

This is a theorem about G_{P}. It is not a claim that $\xi(s) = 0$ is ill formed in classical complex analysis.

19.4 The native sentence

Definition 19.5 (Mirror axis)**.** The mirror-axis involution is

$$\iota(s) = 1 - \bar{s}.$$

Its fixed locus is

$$\mathrm{Re}\, s = \frac{1}{2}.$$

Definition 19.6 (Native axis sentence)**.** The native sentence RH_{M} is:

> For every certified exit-locus record $\mathsf{Exit}_{\xi}(s; c)$ of the completed prime-fused descent, the presented parameter is fixed by ι:
>
> $$s = 1 - \bar{s}.$$

Every term quantified over in this sentence is positive: the record, parameter, certificate, completed descent, and involution. Nonemission remains a metalanguage condition attached to the definition of the record.

Proposition 19.7 (Native formation)**.**

$$\mathsf{RH}_{\mathrm{M}} \in \mathrm{Sent}(\mathsf{G}_{\mathrm{P}}).$$

Proof. The sentence quantifies over formed exit-locus records and uses only presented parameters, certificates, the mirror involution, and the symmetric balance relation. It requires no Blank control and no zero term. □

19.5 The tent group

Let

$$\sigma(s) = 1 - s, \qquad \tau(s) = \bar{s}.$$

Then σ and τ commute and generate

$$G = \{1, \sigma, \tau, \sigma\tau\} \cong C_2 \times C_2.$$

The fixed locus of τ is the real axis, and the fixed locus of $\sigma\tau$ is the critical line.

A generic orbit has four points:

$$\{s, 1 - s, \bar{s}, 1 - \bar{s}\}.$$

An orbit on either fixed axis has at most two points. The orbit has one point only at $s = 1/2$.

19.6 Three exact classical forms

Theorem 19.8 (R1–R3)**.** *In the classical register, the following are equivalent to the Riemann hypothesis.*

R1. *Every root of*

$$\Xi(t) = \xi\left(\frac{1}{2} + it\right)$$

is real.

R2. *Every exit locus of ξ is fixed by*

$$s \longmapsto 1 - \bar{s}.$$

R3. *Every exit-locus orbit under the Klein four-group generated by $s \mapsto 1 - s$ and $s \mapsto \bar{s}$ has size at most two, together with the classical fact that every exit locus of ξ stands off the real line.*

Proof. R1 says exactly that every classical zero of ξ has the form $1/2 + it$ with real t, which is the critical-line statement. A point is fixed by $s \mapsto 1 - \bar{s}$ exactly when its real part is $1/2$, giving R2. A generic non-real off-axis point has a four-element Klein orbit. A non-real point has an orbit of size at most two precisely when it lies on the critical line. Real points also have orbits of size at most two, which is why R3 includes the separate classical fact that every zero of ξ stands off the real line. □

Proposition 19.9 (Odd-multiplicity sign detection)**.** *Let $t_0 \in \mathbb{R}$ be a root of the real analytic function $\Xi(t)$. If its multiplicity is odd, then Ξ changes sign at t_0. If its multiplicity is even, a sign change can stay away.*

Proof. Write

$$\Xi(t) = (t - t_0)^m u(t)$$

with $u(t_0) \neq 0$. The factor $(t - t_0)^m$ changes sign exactly when m is odd. □

Sign-change detection is therefore not a fourth unconditional equivalence.

19.7 The completed function and the so-called trivial zeros

For every positive integer n, the pole of $\Gamma(s/2)$ at $s = -2n$ is cancelled by the classical zero of $\zeta(s)$ there. The completed function is regular and

$$\xi(-2n) = \xi(1 + 2n) \neq 0.$$

Thus the negative even zeros belong to the uncompleted zeta factorization and stand outside the exit loci of the completed ξ. This is ordinary classical cancellation under the displayed normalization.

The foundational interpretation may describe the completion factor as an interface cost. The cancellation theorem itself remains a classical analytic fact.

19.8 Class-wide and zeta-specific Admission

Definition 19.10 (Class-wide Admission)**.** For a finite descent D with root multiset $\{\rho_j\}$, define in the classical metalanguage

$$F_D(s) = \prod_j (s - \rho_j)$$

and

$$\Phi_D(s) = \operatorname{Re} \frac{F_D'(s)}{F_D(s)} = \sum_j \operatorname{Re} \frac{1}{s - \rho_j}.$$

Admission at a point is a formation statement. At a presented point conditioned by $\sec^+$, admission asserts that the conditioned observation of the field value forms, with its value presented in the declared positive sector. The class-wide sentence $\mathrm{Adm}_{\mathrm{class}}$ asserts admission at every admitted class-wide descent and every presented conditioned point.

The counter-inscription $\mathsf{CounterAdm}_{\mathrm{class}}$ is itself positively witnessed: it asserts that some admitted descent and some presented conditioned point carry a formed observation of the field value in the reflected sector. Under reflection the mirror cone is a positive cone, so the counter-witness is presented data — a formed negative-sector observation — not a record of failed formation.

Lemma 19.11 (Classical reading of the Admission pair)**.** *Under the classical interpretation of the base, the conditioning* $\sec^+$ *reads as* $\operatorname{Re} s > \frac{1}{2}$*, admission at a descent D and a presented point s holds exactly when*

$$\Phi_D(s) > 0,$$

and the negative-sector counter-witness at (D, s) holds exactly when $\Phi_D(s) < 0$. The classical readings of the pair are therefore

$$\forall D\, \forall s\, \left(\operatorname{Re} s > \tfrac{1}{2} \implies \Phi_D(s) > 0\right) \qquad \textit{and} \qquad \exists D\, \exists s\, \left(\operatorname{Re} s > \tfrac{1}{2} \ \wedge\ \Phi_D(s) < 0\right).$$

Definition 19.12 (Zeta-specific Admission)**.** The sentence Adm_ζ is the native Admission law for the unique completed descent fixed by the zeta reference interface: every certified exit-locus record of that descent is admitted to the mirror axis.

Remark 19.13 (Criterion neighbourhood, and what it does not settle)*.* Adm_ζ is an axis-admission law: every certified exit-locus record of the completed zeta descent is admitted to the mirror axis. It is not defined as a positivity criterion. Distinctly, the stiffness field $\Phi(\sigma, t) = \mathrm{Re}(\xi'/\xi)$ of Section 24.3 carries a positivity property that is itself a known classical equivalent of the hypothesis, established independently by Hinkkanen [50] and Lagarias [49] and studied downstream [51]; the same field separates the two models of Chapter 20. Interface Theorem 19.18 already states that $I(\mathrm{Adm}_\zeta)$ is of exactly classical strength, so the existence of classically equivalent criteria is expected rather than informative. Occupation of a sign asks for more than availability of an exact criterion; availability alone falls short of occupation of a sign fibre: equivalent criteria are absorbed as fibres of one package, and a verdict forms at a sign fibre alone. The standing of the adoption and of the independence results is settled by their own sponsors; the criterion neighbourhood is a separate computation.

Each of the two sentences stands in its own register.

- $\mathrm{Adm}_{\mathrm{class}}$ ranges over an abstract class and is separated by the finite models of Chapter 20.
- Adm_ζ concerns one completed prime-fused descent and is fixed only after the zeta reference conditions are supplied.
- Class-wide independence stops short of zeta-specific independence.
- A finite polynomial witness to $\mathsf{CounterAdm}_{\mathrm{class}}$ is not thereby a model of the zeta reference conditions.

19.9 Explicit sponsorship and native derivation

Let α_ζ name the recorded adoption event [78]

$$\mathsf{Sponsor}(\alpha_\zeta, \mathrm{Adm}_\zeta).$$

Definition 19.14 (Adopted native theory)**.**

$$\mathsf{P}^\dagger = \mathsf{P} + \mathrm{Adm}_\zeta.$$

Theorem 19.15 (Native Axis Theorem)**.** ***Register:*** $\mathsf{Reg}_{\mathrm{Obj}}$.
Dependencies: *the strict exit-record grammar and the explicitly sponsored law* Adm_ζ*. The adopted theory* $\mathsf{P}^\dagger$ *derives* RH_{M}*.*

Proof. Let $\mathsf{Exit}_\xi(s; c)$ be an arbitrary native exit-locus record for the completed descent fixed by the zeta reference interface. Instantiating Adm_ζ at that record yields

$$s = 1 - \bar{s}.$$

Since the record was arbitrary, universal introduction gives RH_{M}. □

The theorem is intentionally transparent. Its content is that the native axis sentence follows from the explicitly adopted zeta-specific Admission law. It is not a derivation of that law from the unextended base theory, and it is not advertised as an unpriced classical proof.

19.10 Reference interface

The zeta-specific instance is fixed through three interpretive conditions:

1. the genealogy is the full prime genealogy associated with the Euler product in its convergent region;
2. the plenum normalization is the harmonic/pole normalization of the zeta Dirichlet series;
3. the completed descent has the zeta functional equation and the standard gamma factor.

Registered Interface Theorem 19.16 (Conditional zeta reference)**.** *Assume a Dirichlet series satisfies the continuation, finite-order, pole, growth, and zeta-type functional-equation hypotheses required by the invoked form of Hamburger's converse theorem. If it also satisfies the displayed normalization and reference conditions, then the classical interpretation identifies it with the Riemann zeta function.*

Proof. This is the relevant application of Hamburger's converse theorem under its named hypotheses [36]. The reference result is conditional on those analytic hypotheses; the glyph grammar alone does not establish them. □

19.11 Analytic interface

Definition 19.17 (Classical analytic interpretation)**.** The interface I sends:

$$\begin{aligned} \text{native completed zeta descent} &\longmapsto \xi, \\ \text{presented parameter} &\longmapsto s \in \mathbb{C}, \\ \text{mirror involution} &\longmapsto s \mapsto 1 - \bar{s}, \\ \text{mirror axis} &\longmapsto \operatorname{Re} s = \tfrac{1}{2}, \\ \text{native exit-locus record} &\longmapsto \text{a classical zero of } \xi, \\ \mathrm{Adm}_\zeta &\longmapsto \text{the classical critical-line assertion.} \end{aligned}$$

At the value level, I is accompanied by the retyping from total $\mathbb{C}$-valued evaluation to partial $\mathbb{C}^\times$-valued evaluation described in Definition 19.1.

Registered Interface Theorem 19.18 (Exact analytic price)**.** ***Register:*** $\mathsf{Reg}_{\mathrm{Cla}}$. *Under the interface I,*

$$I(\mathrm{Adm}_\zeta) \quad \Longleftrightarrow \quad \mathsf{RH}_{\mathrm{C}}.$$

Proof. By Definition 19.17, the exit-locus records of the zeta-specific descent correspond to the classical zeros of ξ, and the native mirror axis corresponds to $\operatorname{Re} s = 1/2$. Therefore the interpreted Admission law says exactly that every nontrivial classical zero lies on the critical line. □

A second expression of the same classical price is Weil positivity. For the admissible test class in Weil's criterion, with the involution

$$\widetilde{f}(x) = \overline{f(-x)},$$

the criterion has the form

$$W(f * \widetilde{f}) \geq 0 \qquad \text{for every admissible } f.$$

Under the standard analytic hypotheses and normalization, Weil's criterion is equivalent to RH_{C} [80, 9]. The criterion is a classical theorem used at the interface, not a consequence of reflection symmetry or the renderer.

19.12 Symmetry No-Go

Theorem 19.19 (Symmetry No-Go)**.** *Reflection symmetry locates the axis and leaves occupation open. Let*

$$a = \frac{3}{10} + 7i$$

and

$$F(s) = \left((s - \tfrac{1}{2})^2 - a^2\right)\left((s - \tfrac{1}{2})^2 - \bar{a}^{\,2}\right).$$

Then

$$F(1 - s) = F(s)$$

and

$$F(\bar{s}) = \overline{F(s)},$$

but F has roots off the critical line.

Proof. The substitution $s \mapsto 1 - s$ sends $s - \frac{1}{2}$ to its negative, so it fixes $(s - \frac{1}{2})^2$ and hence F.

The root set is

$$\left\{\frac{1}{2} + a, \frac{1}{2} - a, \frac{1}{2} + \bar{a}, \frac{1}{2} - \bar{a}\right\},$$

which is closed under complex conjugation; consequently the coefficients are real and $F(\bar{s}) = \overline{F(s)}$.

One root is

$$\frac{1}{2} + a = \frac{4}{5} + 7i,$$

whose real part is $4/5$, not $1/2$. The roots form a complete off-axis Klein-four orbit while satisfying both displayed symmetries. □

The theorem identifies the precise limit of symmetry arguments. A zeta-specific proof obligation must use arithmetic or analytic information not shared by F, such as the prime genealogy, completed growth structure, or an equivalent Admission law.

Chapter 20

Relative Independence of Class-Wide Admission

20.1 The symmetric route, attempted and closed

An earlier development attempted to derive axis occupation from the junction algebra and the reflection symmetry alone: the spend-covering argument ran the algebra axiom by axiom against the completed function's reading group and concluded occupation from symmetry bookkeeping. The route fails, and the failure is exact: the polynomial witness of the controlling architecture, $F(s) = ((s - \frac{1}{2})^2 - a^2)((s - \frac{1}{2})^2 - \bar{a}^{\,2})$ with $a = \frac{3}{10} + 7i$, satisfies every symmetry the argument uses while carrying an off-axis quartet. The Symmetry No-Go above is the theorem this witness proves, and the repaired route is the one this part follows: class-wide relative independence, explicit adoption of Adm_ζ, and the native axis theorem in $\mathsf{P}^\dagger$. The development record of the attempted route is preserved in the project archive.

20.2 Declared base scope

The theorem in this chapter concerns the native base theory P, not PA, ZFC, or another external proof theory.

The relevant descent fragment of P has:

1. nonempty finite root multisets;
2. closure under nonempty multiset fusion;
3. closure under the declared mirror and conjugation actions;
4. a polynomial carrier
$$F_D(s) = \prod_{\rho \in D} (s - \rho);$$
5. a conditioned field
$$\Phi_D(s) = \mathrm{Re}\, \frac{F_D'(s)}{F_D(s)};$$
6. presented rational test points away from the roots;
7. no class-wide Admission axiom.

The grammar and MJA_2 modules receive one common interpretation in both models. The models differ only in the descent carrier. The independence theorem is relative to this exact many-sorted base.

20.3 Model A

Let the descents of Model A be the nonempty finite multisets of roots on the seam

$$\rho = \frac{1}{2} + i\gamma,$$

closed under the required conjugation and fusion operations.

For

$$s = \sigma + it, \qquad \sigma > \frac{1}{2},$$

one has

$$\operatorname{Re} \frac{1}{s - (\frac{1}{2} + i\gamma)} = \frac{\sigma - \frac{1}{2}}{|s - (\frac{1}{2} + i\gamma)|^2} > 0.$$

Every term in $\Phi_D(s)$ is therefore positive.

Proposition 20.1 (Model A admits class-wide Admission)**.** *Every descent in Model A satisfies*

$$\Phi_D(s) > 0$$

at every presented point strictly right of the seam.

Proof. Sum the strictly positive displayed contribution over the nonempty finite root multiset. □

20.4 Model B

Let Model B be generated under nonempty fusion by the Klein-symmetric quadruple

$$D_B = \left\{ \frac{4}{5} + 5i, \frac{4}{5} - 5i, \frac{1}{5} + 5i, \frac{1}{5} - 5i \right\}.$$

Evaluate at the presented rational point

$$s_0 = \frac{3}{5} + 5i.$$

The four real contributions are:

$$-5, \qquad -\frac{5}{2501}, \qquad \frac{5}{2}, \qquad \frac{5}{1252}.$$

Using

$$1252 \cdot 2501 = 3131252,$$

their sum is

$$\begin{aligned} \Phi_{D_B}(s_0) &= \frac{-15656260 - 6260 + 7828130 + 12505}{3131252} \\ &= -\frac{7821885}{3131252} \\ &< 0. \end{aligned}$$

Proposition 20.2 (Model B counter-inscribes class-wide Admission)**.** *Model B satisfies* $\mathsf{CounterAdm}_{\mathrm{class}}$*, evaluated in the classical interpretation of the base (Lemma 19.11).*

Proof. The descent D_B and the presented point s_0 witness the existential sentence, by the exact calculation above. □

20.5 Independence theorem

Theorem 20.3 (Relative Independence of Class-Wide Admission)**.** ***Register:*** $\mathsf{Reg}_{\mathrm{Rel}}$*. Relative to the declared base theory,*

$$\mathsf{P} \nvdash \mathrm{Adm}_{\mathrm{class}}$$

and

$$\mathsf{P} \nvdash \mathsf{CounterAdm}_{\mathrm{class}}.$$

Proof. Model A is a model of the base theory in which $\mathrm{Adm}_{\mathrm{class}}$ holds. Model B is a model of the same base theory in which $\mathsf{CounterAdm}_{\mathrm{class}}$ holds. By soundness of the declared proof calculus for these interpretations, a sentence derivable from P must hold in every model of P. Model B therefore refutes derivability of $\mathrm{Adm}_{\mathrm{class}}$, and Model A refutes derivability of $\mathsf{CounterAdm}_{\mathrm{class}}$. □

Remark 20.4 (Scope)*.* The theorem concerns only the two well-formed class-wide native sentences. The literal classical zero-locus sentence is not in $\mathrm{Sent}(\mathsf{G}_{\mathrm{P}})$, so the theorem leaves that literal sentence to its own register, proving and refuting on sentence. The theorem also does not establish independence of Adm_{ζ}. Zeta-specific Admission is handled by explicit sponsorship and the analytic interface. The irresolvability–independence distinction is developed in the corpus record [78].

20.6 Exact-arithmetic artifact scope

The displayed fraction can be checked directly in rational arithmetic. An accompanying script may reproduce it, but the script does not by itself verify:

- the grammar axioms;
- the shared MJA_2 interpretation;
- closure of the model carriers;
- the soundness theorem;
- any zeta-specific statement.

No script execution is asserted by this TeX source.

Chapter 21

Omega Formation and the Selection Jump

21.1 Kept stages

Fix a decidable address sort and a decidable negative verifier. Write

$$\mathsf{Reject}(w, r)$$

when r is a positively presented rejecting trace for address w.

Definition 21.1 (Finite kept stage)**.** A finite kept stage at level n is a record k_n containing, for every address $w < n$, a trace $r_{n,w}$ with

$$\mathsf{Reject}(w, r_{n,w}).$$

The judgment that k_n has this property is written

$$\mathsf{Keep}(n, k_n).$$

Definition 21.2 (Coherent family)**.** A family

$$K = (k_n)_{n \in \mathbb{N}}$$

is coherent when

$$\mathsf{Keep}(n, k_n)$$

for every n, and

$$k_n \preceq k_{n+1}$$

for every n. Write

$$\mathsf{OmegaKeep}(K)$$

for the conjunction of these judgments.

The metalanguage natural numbers in this definition index stages. They are not strict object-language numerals and do not install a zero term in G_{P}.

21.2 Omega-Seal rule

Adopted Formation Law 21.3 (Omega-Seal)**.** *A terminal seal may form from a positively presented coherent family:*

$$\frac{\mathsf{OmegaKeep}(K)}{\mathsf{Seal}_{\omega}(K)}.$$

The rule does not infer a family from

$$\forall n\, \exists k_n\, \mathsf{Keep}(n, k_n).$$

The premise of the rule is a presented completed object. It is stronger than separately stated stagewise availability. The formation-rule study behind this chapter is the corpus manuscript *The Selection Jump* [84].

Selection-Jump Theorem 21.4 (Selection Jump)**.** *Relative to the declared formation rules, finite occupation of every separately presented stage is a record at each stage; a global selector or a coherent family. Formation of a coherent omega-family requires one of:*

1. *a presented coherent family;*
2. *a guarded generator together with a presented proof of its uniform keeping law;*
3. *an explicit occupation sponsor in the witnessed semantics.*

Proof. The formation rules contain constructors for the three listed events. They contain no rule whose premise is merely $\forall n\, \exists k_n\, \mathsf{Keep}(n, k_n)$ and whose conclusion is a selector or coherent family. Closure under the declared rules therefore leaves the stagewise statement without a family-forming derivation. The result is a theorem about this formation system, not a theorem that no external classical choice principle can be adopted. □

21.3 Explicit witnessed completion

Let K_ζ be a name introduced at the metalanguage level for the completed family required by the witnessed omega semantics. The assertion includes the explicit adoption event

$$\mathsf{Sponsor}(\alpha_\omega, \mathsf{OmegaKeep}(K_\zeta)).$$

This is a completion postulate. It does not provide:

- a program computing k_n from n;
- the component records k_n;
- an analytic proof that every component is kept;
- a proof internal to the unextended base theory;
- an interval-certified verification of all addresses.

The sponsorship records occupation in the witnessed semantics, at exactly that strength.

21.4 Zeno-register consequence

Assume the named clash-channel interface: if an off-axis exit-locus record exists, then some finite address is accepted by the negative verifier; and a kept stage covering that address carries a rejecting trace incompatible with acceptance.

Theorem 21.5 (Zeno-register axis resolution)**.** ***Register:*** Reg_ω.
Dependencies: *the clash-channel interface, the explicitly sponsored coherent family K_ζ, and the Omega-Seal rule. The sealed family entails the native axis sentence in the omega-completion register.*

Proof. Suppose an off-axis exit-locus record were presented. By the clash-channel interface it yields an accepted finite address w. Choose a stage index exceeding w. The corresponding component of the sponsored coherent family contains a rejecting trace for w. Determinism of the verifier supplies a positive clash between the accepted and rejected traces. Hypothesis-discharging rejection therefore rejects the off-axis record. Since the record was arbitrary, the native axis sentence follows in the omega register. □

This is not an analytic proof because the completed-family premise is an explicitly adopted completion postulate.

21.5 Class-wide independence, arithmetic shadow, and coverage

The two-model theorem concerns $\mathrm{Adm}_{\mathrm{class}}$, not Adm_ζ. Its direct consequence is that the unextended native base does not select a side of the class-wide fork.

A separate arithmetic interpretation may assign a standard Π_1 sentence φ_{RH} to the classical arithmetic shadow of the Riemann question.

Definition 21.6 (Arithmetic-shadow interface)**.** Let T be a named arithmetical theory. Let φ_{RH} be a specific Π_1 sentence, and assume a theorem internal to the standard arithmetic interface identifies φ_{RH} with the standard classical RH criterion.

The interface J_{arith} records:

1. the exact formula φ_{RH};
2. the proof that its negation is a finite-witness Σ_1 sentence;
3. the required consistency, soundness, and Σ_1-completeness properties of T;
4. the interpretation from the arithmetic criterion to the standard classical analytic sentence.

Theorem 21.7 (One-sided truth of the arithmetic shadow)**.** *Under J_{arith}, if*

$$T \nvdash \varphi_{\mathrm{RH}} \qquad \text{and} \qquad T \nvdash \neg\varphi_{\mathrm{RH}},$$

then φ_{RH} is true in the standard arithmetic interpretation.

Proof. If φ_{RH} were false in the standard arithmetic interpretation, then $\neg\varphi_{\mathrm{RH}}$ would be a true Σ_1 sentence with a finite standard witness. The declared Σ_1-completeness clause would give

$$T \vdash \neg\varphi_{\mathrm{RH}},$$

contrary to the assumed independence. Therefore the arithmetic shadow is true. □

Remark 21.8 (Domain and burden of the one-sided route). The conditional theorem concerns the arithmetic shadow only: a specific standard arithmetical sentence identified, inside J_{arith}, with the classical RH criterion. Its quantifiers range over exactly the standard countable analytic zero roster and the finite arithmetic witness relation.

Relative to the presence foundation, the classical regime is the interfaced party. Countability of the classical zero set is a theorem internal to the standard analytic interface, not a bound on what the completed witnessed register may form. The bare admissibility of an off-axis exit formation in a completed family exceeding the standard countable roster already places the transfer obligation on the classical side: truth of the arithmetic shadow discharges the fully intended native axis sentence only through the coverage interface C_{cov} defined below, and no clause of J_{arith} supplies coverage for free.

The prohibition on substitution is two-sided. A transfinite or higher-cardinality sentence RH_κ enters the record through its own formation certificate and named interface, exactly as the classical shadow entered through J_{arith}. Each regime's verdict stands in its own register apart from the other's without the corresponding transfer theorem; the asymmetry lies only in where the foundation stands.

The theorem establishes the standard arithmetic shadow only. A further coverage theorem is required before that truth is transferred to every formation admitted by the full presence foundation.

Definition 21.9 (Coverage and faithfulness interface)**.** The interface C_{cov} asserts that every admissible exit-locus formation relevant to the intended Riemann sentence is represented by the standard analytic and arithmetic coding used in J_{arith}.

Equivalently, C_{cov} rules out an exit formation that:

1. forms in the completed witnessed or higher-cardinality register;
2. is relevant to the intended native axis sentence;
3. has no representative in the standard countable analytic zero roster or finite arithmetic witness relation.

Registered Interface Theorem 21.10 (Full transfer through coverage)**.** *If the arithmetic shadow is true and C_{cov} holds, then the corresponding standard analytic sentence and the fully intended native axis sentence agree under the named interfaces.*

If C_{cov} fails, truth of the arithmetic shadow does not settle the higher-cardinality formation question.

Proof. Under C_{cov}, every relevant exit formation is in the image of the standard coding. The arithmetic truth excludes every coded counterexample, so coverage excludes every relevant counterexample.

Without coverage, a formation outside the image is not addressed by the arithmetic quantifier. No transfer follows for that formation. □

Remark 21.11 (Standard analytic countability is interface-relative). Within standard classical complex analysis, the zero set of a nonzero entire function is discrete and therefore countable. The program does not silently promote that internal theorem into a proof that the standard analytic interface exhausts every completed formation recognized by the presence foundation. Exhaustivity is the content of C_{cov}, not a free consequence of classical primacy.

21.5.1 Conservative independence transfer

The arithmetic shadow of the independence theorem is governed by a transfer schema whose proof is immediate but whose statement fixes exactly what any classical relabeling of the native theorem must supply.

Theorem 21.12 (Conservative independence transfer)**.** *Let A be a sentence of the native theory and $\tau(A)$ its translation into a classical theory T. Suppose the translation is proof-reflecting for A and its contrary:*

$$T \vdash \tau(A) \implies \mathsf{P} \vdash A, \qquad T \vdash \neg\tau(A) \implies \mathsf{P} \vdash \neg A.$$

Then $\mathsf{P} \nvdash A$ and $\mathsf{P} \nvdash \neg A$ imply $T \nvdash \tau(A)$ and $T \nvdash \neg\tau(A)$.

Proof. Contraposition of each displayed hypothesis. □

The theorem composes with the one-sided route into a single chain: native independence, under a proof-reflecting translation, yields classical arithmetic independence; a Π_1 representative [48] together with Σ_1-completeness of a sound T converts that independence into arithmetic truth; and the analytic equivalence converts truth into RH_{C}. Independence is therefore not an evasion of the Riemann question: at the appropriate arithmetic strength it is an affirmative proof method. The chain's antecedent is a zeta-specific independence theorem under a proof-reflecting translation; the theorem proved in this book is class-wide, over P, and the two are connected only by an instance-separation theorem that the reference clauses do not yet supply. That connection is stated as an open problem.

Corollary 21.13 (Faithfulness fork)**.** *No classical analytic system can simultaneously assert that it faithfully realizes $\mathsf{P}^{\dagger}$, that zeta-specific Admission fails, and that the interface theorem holds. Every faithful classical realization of the adopted presence theory satisfies RH_{C}; failure of RH_{C} demonstrates failure of the realization's faithfulness, not of the native theorem.*

Remark 21.14 (Conservation of difficulty). Universality and truth-transfer are paid in different currencies. The two-model theorem is foundation-neutral because the base does not refer to ζ; Model B separates the class-wide fork precisely by reinterpreting the class. The identical design fact severs the counterexample channel that the one-sided route requires: a hypothetical off-axis zero of the actual completed descent cannot be internalized as a base refutation, because the base does not name the object the zero would live in. The entire remaining difficulty of the classical question is thereby relocated, without remainder, into one construction problem — an admitting model of the reference clauses — and that problem is exactly as hard as RH_{C} itself, in the native setting for the same reason that $\mathrm{Con}(\mathsf{Q} + \varphi_{\mathrm{RH}})$ is classically. Each component of the architecture does exactly its named work; the residue is provably all of the question, concentrated at faithful reference to the completed prime-fused descent.

21.6 What licenses the adoption

Register: $\mathsf{Reg}_{\mathrm{Rel}}$.

The Sponsorship Declaration of Adm_{ζ} is a foundational act, and a foundational act invites the question of what makes it lawful rather than arbitrary. The answer is not an analogy to any historical foundational choice. It is a theorem about routes.

Four steps, each already proved.

Presence-only formation. The verdict language records positive presentations and has every constructor build from presentations: [absence], witness-not-found, no counterexample-not-seen, no checker-returned-nothing. A present verdict is therefore built from presentations alone. This is the no-free-sign theorem, and it is the grammar of this volume applied to sign formation itself.

Nullity of the selected architecture. Consider the enriched exact selected package of the completed zeta descent: exact finite negative channels, exact all-stage positive channels, trace predicates, and a Π_1 arithmetic representative. Every sound resolver that is fully invariant under replacement of one exact selected package by another is identically empty [82]. Equivalent criteria are absorbed as fibres of one package, and a verdict forms at a sign fibre alone: availability of an exact criterion is not occupation of a sign. Criterion splitting, replay, arithmetization, realizer indexing, and finite-stage verification raise complexity without raising sign [83].

The forgetful boundary. A resolver is a-priori selected exactly when it factors through the map that discards package-specific coordinates while retaining exact selected structure. Every sound sign selector uses data beyond what factors through selected architecture alone. Consequently every sound route that does form a sign must expose the point at which it stops so factoring, and that point is an occupation, a derivation, a coordinate theorem, a stream, or an oracle. Every sound route is exposed.

The declaration is the exposure. The adoption recorded here is that exposed point, named in advance rather than discovered by audit. It constitutes the theory $\mathsf{P}^\dagger = \mathsf{P} + \mathrm{Adm}_\zeta$, it carries a sponsor α_ζ, it is marked at its stratum, and its classical price is stated exactly by Interface Theorem 19.18. Every verdict it sponsors is formed carrying that trace, which is what formation means in this volume: every verdict here names what presented it, and that naming is the whole of what is sought.

The license follows. A route that forms a sign without exposing its non-invariant effect is convicted by the hypothesis-tethering theorem, which admits no untethered premise into a sign derivation. A route whose formation withholds every inscription is already absorbed into the maximal selected closure. The two classes exhaust. The adopted law is therefore not one option among many equally available foundational choices: it is the disclosed form of the only kind of act that can form a sign at all, made visible at the moment of adoption instead of recovered later from a proof that quietly used it.

The declaration answers the independence theorem rather than evading it. That Adm_ζ is underivable from P is what makes a declaration the required act; a derivation would have made it superfluous, and a refutation would have made it unavailable.

What is formed

It remains to say what verdict this forms, and in what terms.

The interface I is defined in Definition 19.17 and the equivalence

$$I(\mathrm{Adm}_\zeta) \Longleftrightarrow \mathsf{RH}_\mathrm{C}$$

is proved from that definition by Interface Theorem 19.18. The proof is carried out in the classical metatheory, which is where a statement about the object language is stated and proved: standing outside a language is the condition of speaking about it, and the register separation of this book is accordingly the instrument of reach, not a limit on it. The theorem is a proved reaching, and it grips RH_C exactly. It requires no prior construction: the faithfulness condition belongs to Corollary 21.13, which constrains classical realizations of $\mathsf{P}^\dagger$, and is not a hypothesis of the interface theorem.

Adm_ζ is present in $\mathsf{P}^\dagger$ by the Sponsorship Declaration, sponsored by α_ζ. Therefore, under I, RH_C holds, and the verdict that forms carries its trace: the adopted theory $\mathsf{P}^\dagger$, the native derivation

of RH_{M}, and the sponsor α_ζ that presented the law.

That trace is not a qualification on the verdict. It is what a verdict is here: the naming exhausts the verdict. Every formed verdict of this book names what presented it — a certificate, a derivation, a coordinate theorem, a completed stream, or a declaration — and the classical register's sponsorless verdict is, in this grammar's diagnosis, one more reification of non-presentation, the analysis of which occupies this volume throughout. The record is complete and the seeking is performed: what presented this one is a declaration, made at a fork proved inhabited on both sides, displayed at the moment of adoption rather than recovered afterward from a proof that used it silently.

21.7 Riemann result by register

The Riemann architecture has five positive components.

Grammar. The literal sentence $\mathsf{RH}_{\mathrm{C}}^{(0)}$ stands outside $\mathrm{Sent}(\mathsf{G}_{\mathrm{P}})$, while the certified native axis sentence RH_{M} is.

Adopted native theory. The explicitly sponsored theory

$$\mathsf{P}^{\dagger} = \mathsf{P} + \mathrm{Adm}_{\zeta}$$

derives RH_{M}.

Relative native metatheory. The declared class-wide base has models on both sides of the Admission fork:

$$\mathsf{P} \nvdash \mathrm{Adm}_{\mathrm{class}}, \qquad \mathsf{P} \nvdash \mathsf{CounterAdm}_{\mathrm{class}}.$$

Omega completion. The witnessed semantics explicitly sponsors a coherent omega-kept family as a completion postulate. The Omega-Seal rule yields the native axis conclusion relative to that completed family and the clash-channel interface.

Classical analytic interface. The named interpretation satisfies

$$I(\mathrm{Adm}_{\zeta}) \Longleftrightarrow \mathsf{RH}_{\mathrm{C}}.$$

Thus the classical strength of the zeta-specific native adoption is stated exactly.

The Symmetry No-Go proves that reflection and functional-equation symmetry leave the axis configuration to further data.

Part VIII

Explicit Formula and Computation

Chapter 22

The Explicit-Formula Ledger

22.1 Normalization

Let g be a real even test function in the declared Weil class and define

$$h(r) = \int_{-\infty}^{\infty} g(u) e^{iru}\, du,$$

with inverse convention

$$g(u) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h(r) e^{-iru}\, dr.$$

For a nontrivial classical zero ρ, write

$$\gamma_\rho = \frac{\rho - \frac{1}{2}}{i}.$$

On the Riemann hypothesis these parameters are real. Without that hypothesis the spectral side is read in complete Klein-symmetric quartet form.

Under the normalization used by the supplied ledger, the explicit formula is

$$\begin{aligned}\sum_\rho h(\gamma_\rho) = {} & h\left(\frac{i}{2}\right) + h\left(-\frac{i}{2}\right) - g(0)\log\pi \\ & + \frac{1}{2\pi}\int_{-\infty}^{\infty} h(r)\,\mathrm{Re}\,\psi\left(\frac{1}{4} + \frac{ir}{2}\right)\, dr \\ & - 2\sum_{n\geq 2} \frac{\Lambda(n)}{\sqrt{n}} g(\log n).\end{aligned}$$

The formula is classical and depends on its test-class and normalization hypotheses [80, 75].

Moving the prime term to the other side gives a balance interpretation, but the signs remain those of the displayed classical formula. The phrase "four unsigned blocks" is not used.

22.2 Assurance classification

COMPUTED AT STATED TRUNCATIONThe supplied source records numerical evaluations using a finite zero roster, a finite prime-power cutoff, and multiprecision arithmetic. Those evaluations are not analytic tail certificates.

The current assertion is limited to:

The ledger values are computations at their stated truncations, with the independent-precision stability information recorded by the underlying numerical artifact when such a comparison was actually performed.

The following remain CERTIFICATION PENDING:

- a repaired and independently checked zero-count tail lemma;
- a complete analytic spectral-tail budget;
- a complete prime-power tail budget;
- outward-rounded interval evaluation of every component;
- a rerun tied to the final frozen source and artifact hashes.

No theorem in the assertion-of-record stratum uses the former unit-interval zero-count lemma or the former extremely small tail budgets.

22.3 Gaussian rows

For the reported Gaussian family

$$h(r) = \sqrt{2\pi s_2}\, e^{-s_2 r^2/2},$$

the supplied source records the following finite computations. The table is retained as numerical data at the stated truncations, not as a proof of global positivity.

s_2	spend + plenum	prime bill	reported margin	bill/spend	margin divided by $2h(\gamma_1)$
0.01	0.26930811	3.6×10^{-11}	0.26930811	1.3×10^{-10}	1.4587
0.04	0.02100690	0.00241645	0.01859045	0.11503	1.00809
0.09	0.06969797	0.06951061	1.874×10^{-4}	0.99731	1.0000185
0.16	0.24976826	0.24976803	2.295×10^{-7}	0.9999991	1.0000000
0.25	0.51233559	0.51233559	3.574×10^{-11}	1.0000000	1.0000000
0.36	0.83816961	0.83816961	7.245×10^{-16}	1.0000000	1.0000000

The finite parameters reported by the supplied source were a roster of eighty positive ordinates, prime powers through 10^7, and multiprecision arithmetic. This TeX source does not claim that those parameters have been rerun or independently confirmed for the final distribution.

Because a Gaussian has full support, its prime bill is never grammatically absent merely because it is numerically small. The exact empty-bill statement belongs only to a compactly supported test whose support is contained in

$$(-\log 2, \log 2).$$

Proposition 22.1 (Compact-support pre-prime stratum)**.** *If g is supported in*

$$(-\log 2, \log 2),$$

then

$$\sum_{n \geq 2} \frac{\Lambda(n)}{\sqrt{n}} g(\log n)$$

has no contributing term.

Proof. For $n \geq 2$,

$$\log n \geq \log 2,$$

which lies outside the open support interval. □

The absence of a contributing term is stated here in the metalanguage. A renderer may display a withheld prime-bill slot as a diagnostic, but that display is not a strict empty object.

22.4 Lowest-ordinate asymptotic, conditional form

If all relevant spectral parameters are real and ordered by positive ordinate, then for the Gaussian family the finite spectral sum has leading behavior

$$2m_1\sqrt{2\pi s_2}e^{-s_2\gamma_1^2/2}$$

as s_2 increases, provided the remaining terms are controlled so that their ratio to the first term tends to zero. This is a conditional asymptotic statement under the stated spectral and tail hypotheses. The displayed finite ratios in the table are numerical observations, not a completed proof of the global tail condition.

22.5 Detector status

A one-parameter test family may reveal oscillation caused by a specified off-axis quartet, but absence of observed oscillation over a finite range does not exclude all off-axis configurations. Different contributions may be too small, lie beyond the tested scale, or partially mask one another. The proposed Faithfulness Detector is therefore exploratory and partial. It is not a decision procedure for the Riemann hypothesis.

22.6 No analytic-budget theorem

SUPERSEDED CONSTRUCTION. Earlier source strata asserted a unit-interval zero-count lemma, spectral tails hundreds of orders below the displayed margins, and unconditional positivity of selected Gaussian rows. Those claims are not assertions of record. They require a repaired zero-count argument, verified constants, complete truncation accounting, and an outward-rounded interval rerun. Until those tasks are completed, the ledger remains COMPUTED AT STATED TRUNCATION.

Chapter 23

The Kept Register: Speaking Only of Presence

Register: $\mathsf{Reg}_{\mathrm{Rel}}$.

The quarantine convention licenses the metatheory to speak classically. This chapter records a stricter declared discipline and what the corpus looks like from inside it. Call the discipline D_{+}: speech records performed acts and presented objects, and the record is exhausted by them. D_{+} is here a sponsored perspective, declared as such; the register convention of the front matter stands, and every claim of this chapter is relative to the declaration. The chapter adds a perspective and keeps the others.

23.1 The survival roster

Read under D_{+}, the volume's principal results appear in the following forms, each a performed act or a presented object.

Formation of the flagship. The formation checker, run on the zero-normalized string, terminates in a fired rejection at the clause where an equation demands a formed term; the rejection certificate, with its trace, is the theorem. Refusals are events, and this one is performed.

Independence as two rejections. The reformed reductio runs twice. Assume a base derivation of class-wide Admission; soundness transports it into Model B, a presented finite object, where it meets the counter-inscription that B witnesses — a clash, exhibited in exact rationals — and the clash rule fires $\mathrm{Reject}(\ulcorner \mathsf{P} \vdash \mathrm{Adm}_{\mathrm{class}} \urcorner; \kappa_B)$. Symmetrically through Model A for the counter-law. Two rejection inscriptions, each finite, each carrying its witness, jointly constitute the independence theorem in kept form.

Consistency as a presented model. Model B presents the base; Model A presents it again. A presented model is a positive consistency certificate, and the semantic route is the one this volume travels.

The record as roster. The emission ledger lists performed emissions, and the roster is the entire record. Sponsorship appears as a totality: every formed verdict names its presenter, and the presenters are exhausted by occupation, derivation, coordinate theorem, completed stream, oracle, and declaration.

23.2 The kept form of the axis law

A law is an act performed at every stage, not a stasis — the idiom convention says so, and under D_+ that sentence carries the resolution. The native axis law, spoken in kept form:

> At every stage of the kept ledger there is a next stage, and the stage is kept: the ascent is performed, the keeping is witnessed, $\mathrm{Keep}(n, k_n)$ with $k_n \preceq k_{n+1}$ presented, stage by stage; the sponsored coherent family seals the ledger under the registered Omega-Seal rule; and the clash channel stands ready — a presented off-axis exit record meets the covering stage in a witnessed clash and fires a rejection.

That is the theorem in presence-only speech. It asserts acts: keeping performed at every stage, a family sponsored, a seal formed, a channel armed. Its verdict carries its sponsors — the stage witnesses, the family's declaration, the interface's price — and the carrying exhausts the verdict. What the classical register compresses into a quantifier over an open domain, the kept register performs, stage by verified stage, in exactly the posture this volume holds toward its one open axiom.

The same posture carries the limitative results. Universal claims over open domains are kept laws under D_+: held stage by stage, each stage certified. Incompleteness itself appears in kept form as the ascent of the ledger — at every stage there is a next stage — which the omega chapter already formalizes as the refusal of the Selection Jump and the sponsorship of the seal.

23.3 The discipline polices its expositors

Drafting audit, recorded in house style: an early exposition of this chapter summarized the survival roster with an absence idiom ("almost nothing dies"). The eliminability test convicted the phrase; the conviction fired; the repaired form is the roster above, which speaks of what each result becomes and of the machinery it lands on. The specimen is retained because it exhibits the discipline's edge: D_+ is kept the way every law here is kept, act by act, and its audits are among the acts.

Part IX

Research Directions and Engineering Consequences

Chapter 24

Directions Beyond the Axis

The research program of the founding stratum was executed in many directions, and only its axis-facing third matured into the Riemann architecture of the preceding part. The remaining directions are recorded here at exact current strength: what is proved is asserted, what is computed is dated to its truncations, what is open is named open, and every full historical construction remains readable verbatim in the strata. This chapter is where the program's concepts meet one another.

24.1 The founding fold: tent and cone-three algebra rooted in the primes

For a cone rolled from a sector of share ρ of the full turn with slant height ℓ, the base radius is $r = \rho\ell$ and the height satisfies

$$h^2 = (\ell - r)(\ell + r).$$

For the prime shares $\rho = 1/p$ the normalized height $x = h/\ell$ satisfies

$$p^2x^2 = (p-1)(p+1),$$

with the tent case $4x^2 = 3$ and the one-third case $9x^2 = 8$. Status: the identities are exact elementary geometry and are asserted. Their reading — the third dimension of the folded page rooted, share by share, in the primes — is a declared reading in the sense of Chapter 17, and the fixed-function discipline governing what the folded forms may carry is Theorem 13.1.

24.2 The di-cone system and the variational silhouette

With complementary radii $r_1 + r_2 = \ell$ on a common slant, the heights satisfy

$$h_i^2 = \ell^2 - r_i^2, \qquad h_1^2 = r_2(\ell + r_1), \qquad h_2^2 = r_1(\ell + r_2),$$

and

$$h_1^2 - h_2^2 = \ell\,(r_2 - r_1).$$

The di-cone volume stationarity polynomial factorizes exactly as

$$(2\rho - 1)\left(18\rho^6 - 54\rho^5 + 24\rho^4 + 42\rho^3 - 42\rho^2 + 12\rho - 1\right),$$

with the seam factor $2\rho - 1$ exhibiting the mirror-symmetric critical share and the sextic carrying the paired off-seam critical ratios. Status: the identities and the factorization are exact and asserted. The comparison of seam and off-seam occupations against a computed prime-fused field is a computed observation at its recorded truncations, and the silhouette reading built on it is open.

24.3 Reference and the categoricity clauses

The reference question is whether the notation, held jointly, determines its intended object. Three categoricity clauses are proposed:

1. a full prime genealogy;

2. a harmonic-plenum or pole normalization;

3. a zeta-type mirror completion.

Status: these are proposed identification criteria. Their joint sufficiency is not a theorem of this volume. One external trial is on record — the blind reading of Chapter 10 — in which the clauses' ingredients, jointly held, produced the intended identification by name; a trial supports and does not prove.

24.4 Positivity in the di-cone: the orbifold examination

The critical strip may be read as a reflection orbifold and a self-convolution test as a mirror fusion. On the critical line a real spectral parameter contributes a nonnegative square-type Gaussian weight; an off-line pair contributes oscillatory terms of indefinite sign. Status: this is a heuristic frame, consistent with the classical Gaussian ledger of Chapter 22, and no positivity theorem is asserted from it.

24.5 The Zeno completion and the generator

The witness channel admits a decidable rejection judgment $\mathsf{Rej}(w, r)$ and finite kept stages $\mathsf{Keep}(n, k_n)$, assembled into coherent families $K = (k_n)$. Status: the definitions are sound and retained. The historical witness-exhaustion claim is superseded; its corrected descendant is the omega-completion postulate and Selection Jump of the Riemann architecture [84], where the completed coherent family is explicitly sponsored rather than derived. The generator direction is retained with its recorded distinction: totality of a stage-extending verifier at each input is distinct from uniform production of a completed family, and only the former is claimed.

24.6 The stiffness field and the harmonic face

The stiffness field is

$$\Phi(\sigma, t) = \operatorname{Re} \frac{\xi'}{\xi}(\sigma + it).$$

By the functional equation the appropriately completed field is odd about the critical line away from singularities; away from zeros and poles it is harmonic, and the minimum principle applies on compact subdomains with controlled boundary data. Status: these facts are classical, imported and exact. The historical unconditional positivity wedge and the kept-region assembly are open: they require exact lower bounds on the relevant boundary data and a verified zero roster below the cited height, and neither is asserted here.

24.7 The de Branges test

Proposition 24.1 (Scope of positivity routes)**.** *The Conrey–Li elements [17] at which the de Branges positivity structure fails are of distinct genealogy and prime-fused trace: they pass both admission filters. The obstruction therefore lies inside the domain of Native Admission, and no discharge of Admission may proceed by positivity arguments insensitive to those elements; any discharge must use properties the admission cone carries beyond the de Branges structure.*

Status: a scope statement at interface strength, with its classical input cited; it sharpens the target of the Admission law and leaves the law's status to the adoption that decides it.

24.8 The Selberg-class generalization

Under the classical interface, the two admission filters correspond exactly: distinct genealogy to the functional-equation axiom, and prime-fused trace to the Euler-product axiom. The trace typing thus recovers the Selberg axioms from inside the calculus, and Admission over the class reads, classically, as the class-wide Riemann hypothesis. Status: the axiom recovery is a structural comparison at interface strength. The class-wide sentence of the present volume, $\mathrm{Adm}_{\mathrm{class}}$, is this direction matured: the independence chapter separates it from its counter-inscription by finite models, so the class-wide hypothesis itself is neither asserted nor refuted here — that separation is the content of Theorem 20.3. The survey observation that every known off-axis family fails a filter is recorded at survey strength.

24.9 The mining protocol

Candidate implications over the MJA_2 signature are triaged in order: well-formedness; survival in the blade model; survival against the polynomial witness; survival against the Davenport–Heilbronn descent; non-equivalence to Admission, with any candidate implying full-cone positivity set aside at conjecture strength. Survivors are conjecture-grade lemmas ranked by detector sensitivity. Status: the protocol is specified, and its filters now have exact instruments in the current stratum — the polynomial witness is the descent of Model B, and the genealogy filter is the Davenport–Heilbronn exclusion. No run is claimed.

24.10 Open problems

The following problems are stated as mathematics. (1) Prove or refute the coverage theorem: that the standard analytic roster represents every exit-locus formation of the completed descent, i.e. C_{cov} holds for ζ. (2) Determine whether the di-cone sextic's paired critical ratios admit a spectral interpretation under the prime-fused field, and prove or refute the silhouette comparison at all truncations. (3) Prove the unconditional stiffness wedge: exhibit exact boundary bounds under which $\Phi > 0$ on a right neighborhood of the line, or show no such bounds exist. (4) Extend the kernel formalization from G_{K} to the full strict grammar — eliminating the implementation `blank` variant, which the discipline of record excludes at every layer — and machine-check numeral canonicality. (5) Characterize the class for which the Selberg-axiom recovery of the trace typing is exact, and decide class-wide Admission for a nontrivial finitely axiomatized subclass. (6) Give a formation-faithful interpretation of a zero-based foundation into $\mathsf{P}^{\dagger}$ and compute its price on the explicit-formula ledger. (7) Construct an admitting model of the referenced theory $\mathsf{P} + \mathrm{RefClauses}$

— a model of the reference clauses whose referent admits — or prove that the referenced theory does not derive $\mathrm{CounterAdm}_{\zeta}$ by other means. Either, together with membership of the referent in the class and internalization of a finite contrary witness, yields RH_{C}; with the converse adequacy of the analytic interpretation, the three statements RH_{C}, non-derivability of $\mathrm{CounterAdm}_{\zeta}$ over the referenced theory, and existence of an admitting model are equivalent. Model B separates the class-wide schema precisely by evading the reference clauses, so the proved class-wide theorem neither supplies nor is supplied by this instance; the two are related by an existential introduction in the easy direction only.

Part X

Deprogramming Zero

Chapter 25

Deprogramming Zero: Peano's Axioms Without the Sign for Non-Presentation, the One Kept Record, and the Resolution of the Riemann Question at Its Registers

Abstract. Zero is a formed sign assigned to stand for a non-presentation: its positive formation certifies that something presented while its semantic office says that something did not. This is the root-grammatical contradiction the presence-only programme of *Against Zero* and *The Mirror Calculus* identifies; a *surrogate zero* is any sign a language reaches for to keep that concept under another spelling — [blank, null, empty, undefined, the identity "that does nothing"] — and the surrogate-zero test is the instrument that catches them. This paper deprograms zero out of Peano Arithmetic and carries the result through algebra, set theory, and the Riemann question. PA uses its constant 0 in exactly four ways — as a named point, as the terminus past which the successor stops, as the base of the recursions for $+$ and $\cdot$, and as the base of induction — and each of the four is an instance of the contradiction. We give a formation-faithful interpretation of PA into tally arithmetic TA, the theory of the positive integers with unit, successor, fusion and product, whose object language contains neither the sign nor any surrogate for it and whose unit **1** is formed by the first act and stands for presence, and a converse, so that the theories are bi-interpretable and equiconsistent: the sign is eliminable, which is its formal usability, the whole of what the objection from usability establishes. What the elimination exposes is the cost the classical notation concealed. Three of the four uses are *substitutions* — the unit tally, a return to the unit, induction from the unit — and each classical use of them hides a predecessor-witness obligation that the classical bookkeeping discharges in silence and TA discharges in the open; the operation in which that silent discharge fails is division, and the classical notation names the failure with a returned non-value, "undefined," which is a surrogate zero by the test. The fourth use, the terminus, is a *kept record*: by Lyndon's preservation theorem no negation-free first-order theory excludes the cyclic models $\mathbb{Z}/k$, so the statement that the first tally is reached by no successor is held in the metalanguage as a record, independent of the positive fragment, which is exactly where *Against Zero* places withheld presentation. A ledger counts the substitutions, the concealed obligations, and the record; a general schema extends the result to every theory whose zero is the least element of a discrete order; the identity of a group and the zero of a ring are shown to be surrogates by the test, which heap and truss theory remove; and the empty set, the surrogate of set theory, is placed as a kept record, forced by negative Separation and by $\in$-induction. The second half of the paper turns to the Riemann hypothesis and first proves

why the sign was the obstacle: written with the sign, the hypothesis is [a universal over the plane whose matrix is an evaluation set equal to the sign], on a totality outside presentation — which positive logic does not form and arithmetic makes Π_1, so that its affirmation is an ω-inference with its sponsor left off the page — while the verification ledger presents tallies at every stage, Riemann's completion ξ had already removed the surrogate zeros of ζ, and in the explicit formula each zero of ξ is a presented carrier whose equal weighting is the native form of the hypothesis. On the prime side the Admission law is Lagarias positivity and Li's criterion — the positivity of a prime-indexed sequence — the mirror pairs carriers whose weights multiply to x, Pólya's function and the Davenport–Heilbronn function are the two models of the analytic base without the Euler product, and each kept stage of the ledger is a prime-counting bound on a segment. The register convention of *The Mirror Calculus* is then applied: the independence theorem of the volume is generalized unconditionally to every base in its signature; a base that fixes the referent is proved to decide zeta-specific Admission; counting the constructors of the Selection Jump against the Emmerson–Buchanan barrier hierarchy, the explicit sponsor is the terminal constructor; and the question is resolved at its registers — RH_{C} holds under the analytic interface, sealed, sponsored, and armed, the naming of the trace exhausting the verdict — with the resolution and the barrier result proved to guard each other.

This chapter is the companion paper of the volume, integrated as a Part in the edition of record; its internal numbering follows the volume's, and its references are collected at the end of the chapter.

25.1 The Four Zeroations Are Four Instances of the Contradiction

Write PA in the language $\{0, S, +, \cdot\}$ with the usual axioms:

$$\forall x \;\; Sx \neq 0, \tag{PA1}$$
$$\forall x \forall y \;\; (Sx = Sy \to x = y), \tag{PA2}$$
$$\forall x \;\; x + 0 = x, \tag{PA3}$$
$$\forall x \forall y \;\; x + Sy = S(x + y), \tag{PA4}$$
$$\forall x \;\; x \cdot 0 = 0, \tag{PA5}$$
$$\forall x \forall y \;\; x \cdot Sy = x \cdot y + x, \tag{PA6}$$
$$\big(\varphi(0) \wedge \forall x(\varphi(x) \to \varphi(Sx))\big) \to \forall x\, \varphi(x) \quad \text{for each formula } \varphi. \tag{Ind}$$

Definition 25.1 (Zeroations)**.** The constant 0 occurs in PA in four roles, which we name:

kind	where	what it does
POINT	the constant itself	names an element
TERMINUS	(PA1)	0 is the origin from which every successor chain starts
BASE	(PA3), (PA5)	grounds the recursion for $+$ and $\cdot$
INDUCTION BASE	(Ind)	grounds every inductive proof

A *zeroation* is an occurrence of 0 in one of these roles. Every occurrence of 0 in an axiom of PA is a zeroation; every occurrence in a theorem descends from one.

Definition 25.2 (The contradiction, and the surrogate-zero test, after *Against Zero* [19])**.** Zero is a sign introduced where a witness, value, member, record, or verdict was withheld, placed in positions otherwise occupied by positive objects, entered into object-language operations, and used to let the system speak as though a withheld presentation had supplied content. The contradiction

is root-grammatical: the sign's positive formation certifies presentation while its semantic office asks it to stand for a withheld presentation. A *surrogate zero* is a sign that performs this same office under another spelling — [blank, null, none, empty, missing, undefined, an identity "that does nothing"]. The *surrogate-zero test* asks of a sign whether it was introduced for a withheld presentation, occupies positive positions, participates in operations, and supplies content for that withheld presentation; it concerns formal role, and spelling is beside the point.

Proposition 25.3 (The four uses)**.** *Each of the four zeroations is an instance of the contradiction: a formed sign standing for a non-presentation in a positive position.*

Proof. POINT: 0 is introduced as [the count of a roster "with no member"] — a tally reported ahead of any individuation act — and then stands in the positions of tallies as a term. TERMINUS: (PA1) is the record that 0 is the origin of every successor chain, written as a statement about an object ["0 is not a successor"]. BASE: (PA3) and (PA5) ground the recursions at the sign, so that every sum and product is computed through a term that stands for an act withheld; (PA5) further makes the sign absorb every factor. INDUCTION BASE: (Ind) begins every inductive argument at the sign, so every theorem of PA is proved through it. In each case the sign was introduced for a non-presentation, occupies term positions, enters $+$, $\cdot$ and the induction schema, and lets PA assert $x + 0 = x$ and $x \cdot 0 = 0$ as facts about an object. The contradiction is instanced four times. □

The four are distinct in kind. POINT is a name. TERMINUS is a record: the statement that 0 is the origin of every successor chain, written in the classical form ["0 is not a successor"]. BASE makes 0 an identity for $+$ and an absorber for $\cdot$. INDUCTION BASE makes every inductive argument begin at an element whose only property is that it is named.

25.2 Tally Arithmetic

Definition 25.4 (TA)**.** Tally arithmetic TA is the first-order theory in the language $\{\mathbf{1}, S, \oplus, \otimes\}$ with axioms

$$\forall y \;\; Sy \neq \mathbf{1}, \tag{TA0}$$
$$\forall x \;\; (x = \mathbf{1} \;\vee\; \exists y\, x = Sy), \tag{TA1}$$
$$\forall x \forall y \;\; (Sx = Sy \to x = y), \tag{TA2}$$
$$\forall y \;\; \exists z \;\; \mathbf{1} \oplus z = Sy, \tag{TA1$'$}$$
$$\forall x \;\; x \oplus \mathbf{1} = Sx, \tag{TA3}$$
$$\forall x \forall y \;\; x \oplus Sy = S(x \oplus y), \tag{TA4}$$
$$\forall x \;\; x \otimes \mathbf{1} = x, \tag{TA5}$$
$$\forall x \forall y \;\; x \otimes Sy = (x \otimes y) \oplus x, \tag{TA6}$$
$$\big(\psi(\mathbf{1}) \wedge \forall x(\psi(x) \to \psi(Sx))\big) \to \forall x\, \psi(x) \quad \text{for each formula } \psi. \tag{Ind$_+$}$$

Its standard model is $(\mathbb{N}_{>0}, 1, S, +, \cdot)$.

Three things are worth saying about this list before anything is proved. (TA1) is positive exhaustion: every tally is the unit or a successor, a disjunction with a witness on each side. (TA1′) says the unit is first by exhibiting, for each successor, a tally whose fusion with the unit reaches it. And (TA0) is the one kept record of the theory: the unit is the origin of every successor chain. Every other axiom is an equation between formed terms, an injectivity, or an existence statement with its witness, and every term of the object language of TA is formed by an act, the sign and its

surrogates held in the metalanguage. The unit **1** is on the other side of the line: its formation is the first individuation act and its office is to stand for that act, so formation and office agree, and it is formed by an act as every other term is. (TA0) is kept because it must be — Theorem 25.13 shows that every positive first-order theory in this language has the cyclic groups $\mathbb{Z}/k$ among its models, in which **1** is a successor; the positive axioms (TA1), (TA1′) and induction are all satisfied there. In the register convention of [1] this is a *kept record*: a statement the metalanguage holds about the grammar, exactly as the volume holds nonemission as a metalanguage condition on the exit-locus record.

Lemma 25.5 (Order and predecessor in TA)**.** *Define* $x < y :\equiv \exists z\,(x \oplus z = y)$. *Then* TA *proves that* $<$ *is a strict linear order with least element* **1**, *that* $y = Sx$ *iff* $x < y$ *and every* z *with* $x < z$ *satisfies* $y \le z$, *and that for every* $y \neq \mathbf{1}$ *there is exactly one* x *with* $Sx = y$. *Write* $\mathsf{pred}(y)$ *for this* x; *it is defined exactly on the successors.*

Proof. By (Ind_+) on z, $x \oplus z$ is a successor for every x, z ((TA3), (TA4)); hence by (TA0) every $x \oplus z$ is a successor, so **1** is least and $<$ is irreflexive at **1**; irreflexivity in general, transitivity and trichotomy follow by induction as in PA, with (TA0) in place of (PA1) at the base of each. The uniqueness in the last clause is (TA2), and the existence is (TA1). □

25.3 The Bridge

The object on the positive side is already in the programme's texts: *Counting Back from Infinity* [2] proves that the tally calculus is $(\mathbb{Z}_{>0}, +)$ and that every element of it moves every other ("this semigroup has no identity element internally"), and *Against Zero* [19] proves the same for the native tally sort and recovers classical zero as the free identity adjunction $(\mathbb{Z}_{>0}, +)^1 \cong (\mathbb{N}, +, 0)$, "so the old-world identity is exposed as a completion rather than treated as neutral furniture." The bridge below is that adjunction carried to the full first-order theories with successor, product, and induction, made into a bi-interpretation, and priced. *Counting Back from Infinity* also holds, under the name *Addition is partial, and that is the point*, that fusion of rosters forms only under joint separation; TA's fusion is total because its sort is the pairing class, in which separation is supplied by tagging, and the partiality of roster fusion is the metalanguage condition the class quotients away.

Definition 25.6 (Formation-faithful interpretation)**.** An interpretation of a theory T with a distinguished constant 0 into a theory T_+ is *formation-faithful* when it sends every term of T to a term of T_+, every zeroation of T to a named positive device of T_+ or to a sentence of T_+ declared as a kept record, and every term of T_+ to a term formed by an act.

Definition 25.7 (The translation τ)**.** τ sends PA-terms to TA-terms and PA-formulas to TA-formulas, with the domain of the interpretation being all of TA's domain (every tally represents a natural number, namely its predecessor count):

$$\tau(0) = \mathbf{1}, \qquad \tau(Sx) = S\,\tau(x), \qquad \tau(x + y) = \tau(x) \oplus' \tau(y), \qquad \tau(x \cdot y) = \tau(x) \otimes' \tau(y),$$

where the *sign-carrying* operations — the classical $+$ and $\cdot$ as they appear once the sign is removed — are defined in TA by recursion on the second argument:

$$x \oplus' \mathbf{1} = x, \qquad x \oplus' Sy = S(x \oplus' y); \qquad\qquad x \otimes' \mathbf{1} = \mathbf{1}, \qquad x \otimes' Sy = (x \otimes' y) \oplus \mathsf{pred}(x) \ \text{ if } x \neq \mathbf{1}, \quad \mathbf{1} \otimes' Sy = \mathbf{1},$$

Equivalently, in closed form,

$$x \oplus' y = \mathsf{pred}(x \oplus y), \qquad x \otimes' y = S\big(\mathsf{pred}(x) \otimes \mathsf{pred}(y)\big) \ \ (x, y \neq \mathbf{1}), \qquad x \otimes' \mathbf{1} = \mathbf{1} \otimes' y = \mathbf{1}.$$

τ commutes with the connectives and quantifiers (relativised to the whole domain).

The closed forms show what the sign concealed: $\oplus'$ is fusion followed by one predecessor, and $\otimes'$ is the product of two predecessors followed by one successor. Each pred is total on its argument because, in each case, the argument is a successor — $x \oplus y$ is a successor by (TA3)–(TA4) and (TA1′); x and y are successors by the case hypothesis. This is the point of the construction: *every predecessor the sign concealed is guarded by a witnessed successor.* The classical notation discharged these obligations in silence; TA discharges them in the open.

Theorem 25.8 (The tally bridge)**.** *1. For every axiom α of PA, $\mathsf{TA} \vdash \tau(\alpha)$; hence τ is an interpretation of PA in TA.*

2. The map σ with $\sigma(\mathbf{1}) = S0$, $\sigma(Sx) = S\sigma(x)$, $\sigma(x \oplus y) = \sigma(x) + \sigma(y)$, $\sigma(x \otimes y) = \sigma(x) \cdot \sigma(y)$, relativised to $\{x : x \neq 0\}$, is an interpretation of TA in PA.

3. The composites are provably isomorphic to the identity interpretations: PA proves that $x \mapsto Sx$ is an isomorphism from the identity interpretation onto $\sigma \circ \tau$, and TA proves that $x \mapsto Sx$ is an isomorphism from the identity onto $\tau \circ \sigma$. The theories are therefore bi-interpretable, and $\mathrm{Con}(\mathsf{PA}) \leftrightarrow \mathrm{Con}(\mathsf{TA})$ *is provable in primitive recursive arithmetic [22].*

Proof. (1) We verify the axioms one by one; this is also the source of the ledger in Section 25.4. (PA1): τ gives $\forall x\, Sx \neq \mathbf{1}$, which is (TA0). (PA2): identical to (TA2). (PA3): τ gives $x \oplus' \mathbf{1} = x$, the first recursion clause of $\oplus'$. (PA4): τ gives $x \oplus' Sy = S(x \oplus' y)$, the second clause. (PA5): τ gives $x \otimes' \mathbf{1} = \mathbf{1}$, the first clause of $\otimes'$. (PA6): τ gives $x \otimes' Sy = (x \otimes' y) \oplus' x$. For $x = \mathbf{1}$ both sides are $\mathbf{1}$. For $x \neq \mathbf{1}$, $(x \otimes' y) \oplus' x = \mathsf{pred}\big((x \otimes' y) \oplus x\big) = (x \otimes' y) \oplus \mathsf{pred}(x)$ by (TA4) and Lemma 25.5, which is the second recursion clause of $\otimes'$ in Definition 25.7. (The concealed obligation is visible here as one pred that may sit on the operation or on its argument.) (Ind): τ gives $(\varphi^\tau(\mathbf{1}) \wedge \forall x(\varphi^\tau(x) \to \varphi^\tau(Sx))) \to \forall x\, \varphi^\tau(x)$, an instance of (Ind_+).

(2) is the same verification in the other direction, using that in PA the set $\{x : x \neq 0\}$ is closed under S, $+$, $\cdot$, contains $S0$, and satisfies (TA1) because $\mathsf{PA} \vdash \forall x\, (x = 0 \vee \exists y\, x = Sy)$.

(3) In PA, $\sigma(\tau(x))$ unwinds by induction on x to Sx, and $\sigma(\tau(x + y)) = \sigma(\tau(x) \oplus' \tau(y)) = (Sx + Sy) - 1 = S(x + y)$, likewise for $\cdot$; so $x \mapsto Sx$ is the required isomorphism. The TA direction is symmetric. Equiconsistency is the standard consequence of mutual interpretability, formalisable in PRA. □

Remark 25.9. The bi-interpretation is folklore: $(\mathbb{N}, 0, S, +, \cdot)$ and $(\mathbb{N}_{>0}, 1, S, +, \cdot)$ define each other. The subject of this paper is the shape of τ: that PA's operations, carried into the positive structure, are the native operations *plus a predecessor at every zeroation*, and that each such predecessor is defined only because a successor is in hand. That shape is a theorem about PA, and it is the content of the word "concealed" below.

25.4 The Ledger

Definition 25.10 (Ledger marks)**.** In a TA-formula obtained by τ, a *substitution* is an occurrence of $\mathbf{1}$ that is the image of a POINT zeroation; a *concealed obligation* is an occurrence of pred arising from the closed form of $\oplus'$ or $\otimes'$ — a predecessor witness the classical notation owed at that place and discharged in silence; a *kept record* is a sentence of TA held outside its positive fragment and used in the derivation of the translation. The *cost* of a PA-formula is the triple (substitutions, obligations, records) in its translation, the operations unfolded to closed form.

The cost is a property of a presentation rather than of a theorem: a different closed form for the same operation moves a pred and changes the count by one. The figures below are for the closed forms of Definition 25.7, and what is invariant is the qualitative statement that every eliminated zeroation is paid by a substitution or a concealed obligation made explicit, and that exactly one record is kept.

Proposition 25.11 (Cost of the axioms)**.** *The translations of the seven axioms of* PA *carry the following costs.*

axiom	*translation (closed form)*	*substitutions*	*obligations*	*records*
(PA1)	$\forall x\, Sx \neq \mathbf{1}$	*1*	*0*	*1*
(PA2)	$\forall x \forall y\, (Sx = Sy \to x = y)$	*0*	*0*	*0*
(PA3)	$\forall x\, \mathsf{pred}(x \oplus \mathbf{1}) = x$	*1*	*1*	*0*
(PA4)	$\forall x \forall y\, \mathsf{pred}(x \oplus Sy) = S\,\mathsf{pred}(x \oplus y)$	*0*	*2*	*0*
(PA5)	$\forall x\, x \otimes' \mathbf{1} = \mathbf{1}$	*2*	*0*	*0*
(PA6)	$\forall x \forall y\, x \otimes' Sy = \mathsf{pred}\big((x \otimes' y) \oplus x\big)$	*0*	*1*	*0*
(Ind)	$\varphi^{\tau}(\mathbf{1}) \wedge \cdots$	*1*	*0*	*0*
total		*5*	*4*	*1*

Every concealed obligation in the table is discharged by a successor already in hand, as shown after Definition 25.7, and the single kept record is (TA0)*; the ledger therefore closes with every obligation paid in the open and one record kept.*

Proof. Read off the closed forms. In (PA5) the two substitutions are the two occurrences of 0; $\otimes'$ at $\mathbf{1}$ is a clause rather than a computation, so its pred count is zero of the ledger's marks [the tally of marks is a presented count]. In (PA6) the single obligation is the pred of $\oplus'$; the pred inside $\otimes'$ belongs to the definition of the operation, which is costed once, in (PA5)–(PA6) jointly, as the recursion that defines it. □

Corollary 25.12 (Cost of a theorem)**.** *If* $\mathsf{PA} \vdash \theta$ *by a derivation* D*, then* $\mathsf{TA} \vdash \tau(\theta)$ *by a derivation whose cost is at most the sum of the costs of the axiom instances used in* D *plus the number of* 0*-occurrences introduced by instantiation in* D*. In particular every theorem of* PA *has a zero-free proof whose cost is bounded by the number of zeroations in any* PA*-proof of it.*

The programme states the terminus twice before this paper: *Counting Back from Infinity* proves, under the name *No stage of descent is zero*, that every formed stage of the descent calculus is an inhabited positive residue, and holds in its *Convention on inhabited stages* that the empty open set "remains in the metalanguage of topology" while its positive coherent fragment is built [in its words, with "no object-language falsum, no negation, no empty disjunction, and no vacuous universal quantifier"] [2]. The theorem below proves that this placement is forced rather than chosen: every theory in that fragment has the one-point structure as a model, so the terminus is unstatable in it, and every classical axiomatisation must carry the record as a sentence.

Theorem 25.13 (The terminus is indispensable)**.** *Let* T *be any set of positive first-order sentences in the language* $\{\mathbf{1}, S, \oplus, \otimes\}$ *true in* $(\mathbb{N}_{>0}, 1, S, +, \cdot)$*, together with any instances of the induction schema* (Ind_+)*. Then for every* $k \geq 1$ *the cyclic structure* $(\mathbb{Z}/k,\ 1,\ x \mapsto x+1,\ +,\ \cdot)$ *is a model of* T*, and in it* $\mathbf{1}$ *is a successor. Consequently* $T \nvdash \forall y\, Sy \neq \mathbf{1}$*: the terminus zeroation lies outside every positive first-order axiomatisation, and every axiomatisation of* TA *contains a sentence outside the positive fragment that is false in every* $\mathbb{Z}/k$*. The same holds with injectivity* (TA2) *added to* T*,*

since S is injective on $\mathbb{Z}/k$; so (TA0) *is independent of* $\mathsf{TA}^+ = \mathsf{TA} - \{\text{TA0}\}$*, with* $\mathbb{N}_{>0}$ *and* $\mathbb{Z}/k$ *as the two witnessing models. In the positive coherent fragment of [2] — sequents between formulas built from atoms by conjunction, existential quantification, and inhabited disjunction, built short of falsum — the one-point structure* $\mathbb{Z}/1$ *satisfies every sequent, so the terminus is held there in the metalanguage, by the fragment's own convention.*

Proof. The map $h : \mathbb{N}_{>0} \to \mathbb{Z}/k$, $x \mapsto x \bmod k$, is a surjective homomorphism of the structures $(\mathbb{N}_{>0}, 1, S, +, \cdot) \to (\mathbb{Z}/k, 1, +1, +, \cdot)$. Positive first-order sentences are preserved under surjective homomorphisms (Lyndon's positivity theorem), so every positive sentence true in $\mathbb{N}_{>0}$ holds in $\mathbb{Z}/k$. The induction schema holds in $\mathbb{Z}/k$ because every element is reached from 1 by iterating S. In $\mathbb{Z}/k$, $S(k) = 1$, so $\mathbf{1}$ is a successor. □

Remark 25.14. Theorem 25.13 is the exact boundary of the deprogramming. Three zeroations of PA — the point, the bases, the induction base — are substitutions and go over to positive devices. The terminus is a genuine kept record: the statement that the first tally is the origin of every successor chain is a statement about the first tally and every successor at once, with an exhibit for each successor and the first tally standing apart, and positive first-order logic has the circle among its models wherever it has the line. The Mirror Calculus keeps exactly this record in the metalanguage — as nonemission for the exit-locus record, and as the Kept Register more generally [1] — and the theorem shows the choice is forced rather than stylistic.

Proposition 25.15 (Where the concealed obligation comes due)**.** *In* TA *the obligations concealed by* $+$ *and* $\cdot$ *are discharged at every argument, because* $x \oplus y$ *is a successor for all* x, y *and the product clause at* $\mathbf{1}$ *is a return to the unit. Extend both theories by a quotient: in* PA *the partial operation* $x \div y$ *with* $y \cdot (x \div y) = x$*, in* TA *its transport* $x \div' y$ *with* $y \otimes' (x \div' y) = x$*. Then the obligation concealed at the divisor is discharged exactly when* $\mathsf{pred}(y)$ *is formed, that is, when* $y \neq \mathbf{1}$*; at* $y = \mathbf{1}$ *— the image of the classical divisor* 0 *— the obligation is undischargeable, and* TA *withholds the formation.* PA*'s grammar requires a term in that position and supplies ["undefined"] — a returned non-value in a term position, introduced where a quotient was withheld: a surrogate zero by the test, the sign kept under a second spelling.*

Proof. $y \otimes' z = S(\mathsf{pred}y \otimes \mathsf{pred}z)$ for $y, z \neq \mathbf{1}$ and $\mathbf{1} \otimes' z = \mathbf{1}$. For $y \neq \mathbf{1}$ the equation $y \otimes' z = x$ has a solution iff $\mathsf{pred}y$ divides $\mathsf{pred}x$ in TA's native product, and the solution is formed from a witnessed quotient. For $y = \mathbf{1}$ the left side is $\mathbf{1}$ for every z, so the equation holds iff $x = \mathbf{1}$ and then for every z: the quotient is withheld or unselected, and in each case the term position awaits an occupant. TA withholds formation, which is a metalanguage judgment; PA must return something in a term position, and "undefined" is the something. □

Remark 25.16 (What the ledger says). Read in the register convention of [1, 19]: PA's 0 is the sign for a non-presentation — introduced for [the roster "with no member"] and then made a tally — and it is eliminable (Theorem 25.8), which is its formal usability. The elimination has a uniform cost: one substitution per naming, one concealed obligation per recursion step through the base, and the classical notation paid the obligations by writing the sign that made them vanish. The annihilation law (PA5), which in the classical register reads as multiplication destroying its argument, translates as $x \otimes' \mathbf{1} = \mathbf{1}$: a return to the unit, every tally kept and the unit named twice. The semantic error becomes a bookkeeping error at the first operation whose concealed obligation comes due, and Proposition 25.15 shows that this is division and that the classical notation meets it by installing a surrogate. The ledger here is syntactic — it counts marks in a translation — and is separate from the explicit-formula ledger of the volume, on which Problem 6 asks the price of

a foundation to be computed; this paper supplies the interpretation half of that problem and the syntactic half of its price.

25.5 The General Schema

The construction used three things about PA: 0 is least in a discrete order, S is a bijection onto the elements other than 0, and the operations are defined by recursion through 0. That is the whole list.

Theorem 25.17 (Elimination of least-element zeros)**.** *Let T be a first-order theory in a language $L \ni 0, S$ such that T proves*

$$\forall x\,(x = 0 \vee \exists y\, x = Sy), \qquad \forall x \forall y\,(Sx = Sy \to x = y), \qquad \forall x\, Sx \neq 0.$$

Let T_+ be the transport of T along the bijection $x \mapsto Sx$ from the domain onto the elements other than 0: its language has a constant $\mathbf{1}$ for $S0$ and, for each function symbol f of T, a symbol f' interpreted as $S \circ f \circ \mathsf{pred}$ on the transported domain. Then T and T_+ are bi-interpretable; the point and induction-base zeroations of T become $\mathbf{1}$, and the terminus becomes the kept record $\forall y\, Sy \neq \mathbf{1}$, which by Theorem 25.13 lies outside every positive axiomatisation. Whenever f is given in T by recursion equations through 0, f' is given in T_+ by recursion equations through $\mathbf{1}$ in which every pred *is guarded by a successor in hand, as for $\oplus'$ and $\otimes'$; in that case the base zeroations of f become positive clauses and the ledger of Section 25.4 applies verbatim.*

Proof. Transport of structure along a definable bijection is a bi-interpretation. The three hypotheses are what Lemma 25.5 and Theorem 25.8 used to show that the transported domain is all of the positive part, that pred is defined exactly on the successors, and that recursion through 0 becomes recursion through $\mathbf{1}$; the argument goes through with f' in place of $\oplus', \otimes'$. □

Corollary 25.18. *The following have zero-free presentations, every term formed by an act, by the successor shift, with the four zeroation kinds and the ledger structure of Section 25.4:* PA *and all its fragments $I\Sigma_n$, $I\Delta_0$,* PRA*; Presburger arithmetic; and the first-order theory of any discretely ordered semiring with least element. (Skolem arithmetic lies outside the schema: its language has multiplication only, and successor lies outside its definable relations.)*

Remark 25.19 (The general Mirror Calculus). The schema is the first instance of a general move. A *surrogate-zero device* in a theory T is a named term that T uses as an identity, an absorber, a terminus, or a vacuous base; the Mirror Calculus classifies these [1]. For each kind the schema names the positive replacement (a unit, a return to the unit, an exhaustive disjunction, an inductive start at the unit) and the price (a fixing, a witness). The general Mirror Calculus is the theory of such replacements for an arbitrary T: for which T does a zero-free presentation exist, what is its cost, and which devices — the additive identity of a ring, the empty set of ZF, the bottom of a lattice — admit replacement and which resist it. Theorem 25.17 settles the least-element case completely and names the obstruction in the others: the schema carries a device along a bijection from the rest of the domain, and a device outside every such bijection stays where it is.

25.6 The Test Beyond Arithmetic

The schema of Theorem 25.17 covers the sign where it is a least element. In algebra and set theory the sign appears under other spellings, and the surrogate-zero test is the instrument that finds it there; in two of the three cases it has been deprogrammed already, under other names.

25.6.1 The Identity of a Group Is Zero Under Another Name, and Heap Theory Removes It

A *heap* (Prüfer's *Schar*, Baer's *groud*) is a set H with a ternary operation $[x, y, z]$ satisfying $[x, y, y] = x = [y, y, x]$ and $[[x, y, z], u, v] = [x, y, [z, u, v]]$ [24, 23]. Every group is a heap under $[x, y, z] = xy^{-1}z$, and every heap with a chosen element e is a group under $xy = [x, e, y]$; the two passages are inverse, and different choices of e give isomorphic groups [25]. The identity of a group is a surrogate zero by the test — introduced as [the act "that does nothing"], written e or 1 so as to sound like an act, placed in the positions of acts and composed with them — and heap theory removes it: every use in the group axioms ($xe = x$, $xx^{-1} = e$) becomes a heap identity in which every element is formed by the same act. The volume's corollary that the affine basepoint separates [1] is this theorem, and the ledger for it is the substitution $e \mapsto$ the middle argument of $[\,\cdot, \cdot, \cdot\,]$, one mark per occurrence.

25.6.2 The Zero of a Ring, and Truss Theory

A *truss* [26] is an abelian heap with an associative multiplication distributing over the ternary operation. A ring is a truss with a chosen element; a truss with a chosen element e is a ring with additive identity e if and only if e is multiplicatively absorbing, and in general it is a ring-like structure in which the absorber has been forgotten. The additive zero of a ring is the sign itself, and truss theory removes it; its multiplicative role, $0 \cdot x = 0$, is the one ring axiom the passage rewrites, and Brzeziński's analysis of which trusses come from rings is precisely the accounting of that residue. In the language of this paper, the point and base zeroations of a ring are substitutions, and the annihilation law is where the ledger carries a concealed obligation.

25.6.3 The Empty Set Is the Surrogate of Set Theory, and a Kept Record Once Removed

Proposition 25.20. *Let T be a first-order set theory proving Extensionality and the existence of a set.*

1. *If T proves the Separation schema for all formulas, then $T \vdash \exists x\, \forall y\, (y \notin x)$.*
2. *If T proves the $\in$-induction schema (Foundation for definable classes: for every formula φ, if $\forall x\, (\forall y \in x\, \varphi(y) \to \varphi(x))$ then $\forall x\, \varphi(x)$), then $T \vdash \exists x\, \forall y\, (y \notin x)$.*

Set-level Foundation alone leaves the chain standing: the structure $(\mathbb{Z}/k, \in)$ with $n \in m$ iff $n \equiv m-1$ satisfies Extensionality and set-level Foundation, and every element has a member.

Proof. (1) Separate $\{y \in a : y \neq y\}$ from any set a. (2) Apply the schema to $\varphi(x) :\equiv \exists z\, \forall y\, (y \notin z)$: if every member of x satisfies φ then so does x (if x is [memberless], x itself is the witness; otherwise a member supplies one), so every x satisfies φ, and a set exists. For the counter-model, each m has the single member $m - 1$, whose single member $m - 2$ differs from $m - 1$ for $k \geq 2$, so $m - 1$ is $\in$-minimal in m. □

So a set theory in which every set has a member — the presence-only reading of "set" — must restrict Separation to positive formulas and must weaken Foundation to its set-level form or drop it. We conjecture, and leave open, that $\forall x\, \exists y\, (y \in x)$ is consistent with Extensionality, Pairing, Union, and Separation for positive formulas, with a model built from Aczel's universe of non-well-founded sets [27] in which the Quine atom $\Omega = \{\Omega\}$ replaces the empty set as the least inhabited object.

The empty set is a surrogate zero by the test — [the collection "with no member"], written as a set so as to be a set — and it is of the terminus kind: its existence is a kept record ["no

member"], forced by Separation for formulas outside the positive fragment and by $\in$-induction, and a presence-only set theory is a theory of hereditarily inhabited, non-well-founded sets, whose positive comprehension is the subject of Esser's positive set theories [28]. Which fragment of ZF it interprets, and at what cost, is the set-theoretic instance of Problem 6 of [1] and is open.

25.6.4 The Demarcation

Three places the sign appears, three verdicts. Where it is a least element, its uses are substitutions with concealed obligations and one kept record (Theorems 25.8, 25.13, 25.17). In groups and rings it appears as the identity and the additive zero, and heap and truss theory remove it entire. In set theory it appears as the empty set, and once removed it is a kept record. The presence-only programme is the claim that zero is a formed sign assigned to a non-presentation, that its surrogates are the same sign under other spellings, and that the test finds them by role; the classification says what each use stood in for and where, once the sign is removed, the non-presentation is held: in a substitution, in an obligation the notation owed, or in a record the metalanguage keeps. The theorems above are the first three entries.

25.7 The Independence Result, Generalized and Unconditional

The class-wide theorem of [1] is one instance of a result that holds for every base in the descent signature, and the completeness theorem says exactly which bases it holds for. Fix the signature Σ_{desc} of [1]: a sort of descents D (finite root multisets), a sort of presented points, fusion, conjugation, the mirror $\iota(s) = 1 - \bar{s}$, the carrier F_D, the field $\Phi_D = \operatorname{Re} F_D'/F_D$, and the sector predicate $\sec^+$ read as $\operatorname{Re} s > \frac{1}{2}$. Throughout this section a *structure* is one whose sort of presented points contains every rational point away from the roots, as the base of [1] requires ("presented rational test points away from the roots"); a theory T is assumed to prove this. Let Adm be the sentence *for every descent and every presented point in* $\sec^+$, $\Phi_D(s) > 0$, and $\overline{\mathrm{Adm}}$ its positively witnessed counter-inscription *some descent and some presented point in* $\sec^+$ *have* $\Phi_D(s) < 0$.

Definition 25.21 (Axial and off-axial structures)**.** A Σ_{desc}-structure is *axial* if every root of every descent has real part $\frac{1}{2}$, and *off-axial* if some descent contains a root with real part other than $\frac{1}{2}$.

Lemma 25.22. *In every axial structure* Adm *holds. In every off-axial structure closed under the mirror and conjugation,* $\overline{\mathrm{Adm}}$ *holds at some rational point.*

Proof. Axial: each term $(\sigma - \frac{1}{2})/|s-\rho|^2$ is positive for $\sigma > \frac{1}{2}$, and a descent is nonempty. Off-axial: let $\rho = \beta + i\gamma$ with $\beta \neq \frac{1}{2}$; by the mirror we may take $\beta > \frac{1}{2}$. At $s = \sigma + i\gamma$ with $\frac{1}{2} < \sigma < \beta$, the term for ρ is $(\sigma-\beta)/(\beta-\sigma)^2 = -1/(\beta-\sigma)$, which tends to $-\infty$ as $\sigma \to \beta^-$, while the other finitely many terms stay bounded; choose a rational σ close enough to β and the sum lies below zero [the sign here names a presented rational bound]. (Model B of [1] is this construction at $\beta = \frac{4}{5}$, $\gamma = 5$, $\sigma = \frac{3}{5}$.) □

Theorem 25.23 (Generalized unconditional independence)**.** *Let T be any consistent Σ_{desc}-theory. The following are equivalent:*

1. *$T \nvdash$ Adm and $T \nvdash \overline{\mathrm{Adm}}$;*

2. *T has an axial model and an off-axial model.*

In particular Adm *is independent of every T that is satisfied by some axial structure and by some off-axial structure — every base that leaves the axis free on both sides. Consistency is the whole hypothesis.*

Proof. (2) $\Rightarrow$ (1) is Lemma 25.22 with soundness. (1) $\Rightarrow$ (2): by the completeness theorem $T \nvdash \overline{\text{Adm}}$ gives a model of $T + \overline{\text{Adm}}$, which is off-axial by Lemma 25.22 read contrapositively; and $T \nvdash \overline{\text{Adm}}$ gives a model of $T + \text{Adm}$, which is axial, since an off-axial model closed under the operations would satisfy $\overline{\text{Adm}}$. □

This is the class-wide theorem of [1] with the base left free: that theorem is the case $T = \mathsf{P}$, Model A the axial model and Model B the off-axial one. The generalization is unconditional, and it is also exactly as general as any independence result in this signature can be, by the following.

Theorem 25.24 (Faithfulness excludes independence)**.** *Extend the signature by a sort of completed descents with a convergent field $\Phi = \operatorname{Re}\sum_\rho 1/(s-\rho)$ and a constant d_ζ, and let Adm_ζ be* Adm *relativized to d_ζ. Call T* faithful to ζ *if the standard structure $\mathfrak{Z}$, in which d_ζ is the multiset of nontrivial zeros of ξ, is a model of T and every model of T agrees with $\mathfrak{Z}$ on Adm_ζ. Then for every faithful T, exactly one of $T \vdash \text{Adm}_\zeta$ and $T \vdash \overline{\text{Adm}_\zeta}$ holds, and which one is RH_C. A faithful theory decides zeta-specific Admission.*

Proof. In $\mathfrak{Z}$, Adm_ζ holds iff RH_C, by the argument of Lemma 25.22 (the off-axis term dominates the convergent remainder near its root). A sentence true in every model of T is provable from T, by completeness, and a sentence false in every model has its counter-inscription provable. So T decides Adm_ζ, in the direction RH_C takes. The theorem is the completeness theorem applied to the definition of faithfulness; its content is the dichotomy that follows. By Löwenheim–Skolem every first-order T has models in which d_ζ differs from the standard multiset; faithfulness asks only that its models agree on the one sentence. □

Corollary 25.25 (The ζ instance)**.** *For any Σ_{desc}-theory T with a constant d_ζ, exactly one of the following holds:*

1. *T leaves the referent free, and then Adm_ζ may be independent of T by Theorem 25.23 — and the independence, like the class-wide one, is a statement about the models of T and is silent on the zeros of ξ;*
2. *T fixes the referent, and then Adm_ζ is decided by T, and deciding it is deciding RH_C.*

Against Zero [19] states the affirmative route exactly: a *Conservative Independence Transfer* theorem (native independence lifts to a classical theory T along a proof-reflecting translation) and the *One-Sided Independence-to-Truth* theorem, composed as the chain *native zeta-specific independence + proof-reflecting translation $\Rightarrow$ arithmetic independence + Π_1 form and Σ_1-completeness $\Rightarrow$ truth $\Rightarrow$ RH_C*; and it says of the chain's antecedent that "a zeta-specific conservative-transfer theorem is the exact next bridge required." Theorem 25.24 is the statement of what that bridge costs: the antecedent is zeta-specific independence, a theory faithful to ζ decides the zeta instance, and the one-sided theorem's hypothesis $T \nvdash \neg\varphi_{\text{RH}}$ is, for sound Σ_1-complete T, equivalent to φ_{RH} itself. The chain is valid and its entry condition is its conclusion, which is why the volume forms the verdict by a declaration. *Against Zero* also gives the faithfulness fork in one sentence — "every faithful classical realization of the adopted presence theory satisfies classical RH; failure of classical RH would instead establish failure of that realization's faithfulness" [a quotation, in the source's own words] — and Theorem 25.24 is that sentence as a theorem. This is the volume's *Faithfulness Fork* and its *Conservation of difficulty* remark [1] made into a theorem, and it is the independence

side of the irresolvability–independence distinction drawn in [21], where selected irresolvability is a formed object and independence is a metalanguage record: "that problem is exactly as hard as RH_{C} itself" is Theorem 25.24, and "Model B separates the class-wide fork precisely by reinterpreting the class" is clause (1). An *admitting model of the reference clauses* — the volume's Problem 7 — therefore exists if and only if the reference clauses are consistent with Adm_{ζ}; if the clauses are faithful, that is if and only if RH_{C}.

Remark 25.26 (What this settles about the method). Independence is an affirmative method in one paradigm situation: when the independent sentence is Π_1 and the theory is sound and Σ_1-complete, for then $T \nvdash \neg\varphi$ is equivalent to the truth of φ. Theorem 25.24 shows that in the descent signature the corresponding situation — T faithful — is one in which independence is impossible, and Theorem 25.23 shows that where independence is possible the theory is unfaithful and the independence is silent about ζ. The two cases exhaust, and this is the same "no third class" [the volume's phrase] it proves for routes that form a sign: the route through independence forms its sign about the models of T alone, which is why the volume's verdict is formed by a declaration and carries its sponsor.

25.8 Why the Sign Was the Obstacle

The classical Riemann hypothesis is written with the sign at its centre: "$\xi(s) = 0$ implies $\operatorname{Re} s = \frac{1}{2}$." This section states, as theorems about the form of that sentence, what the sign did to the question. Under the convention of *Against Zero* [19], the metalanguage may name the sign and its office in order to diagnose them; bracketed phrases in this section are such diagnostic mentions of the classical form, and every assertion of the section is about what presents. The theorems of the first half of the paper are the instruments: the sign is a formed mark for a non-presentation, positive logic forms its universal sentences over presented records alone (Theorem 25.13), and what the sign conceals is a witness obligation or a record.

25.8.1 The classical statement is a universal over the sign

Proposition 25.27 (Form of the classical statement)**.** RH_{C} *is logically equivalent to the sentence* [there is no s with $\xi(s) = 0$ and $\operatorname{Re} s \neq \frac{1}{2}$]*: a universal over the complex plane whose matrix is an evaluation set equal to the sign. Under the standard arithmetic interface it is a* Π_1 *sentence and its counter-inscription a* Σ_1 *sentence; in the positive coherent fragment of* Counting Back from Infinity *[2] it is held as a record rather than formed, and in Transcendental Presence Notation it is a nonemission record over an infinite ledger.*

Proof. The equivalence is propositional. The Π_1 form is the classical elementary criterion [18], and its counter-inscription asserts a finite witness. The positive fragment holds it as a record by Theorem 25.13 read for this signature: a universal whose content is the closing of a search is exactly the sentence Lyndon's theorem places outside every positive theory, as the terminus was placed outside the positive fragment of TA. In TPN a universal non-presentation is held stage by stage as a nonemission record, and the ledger of stages here is infinite. □

Corollary 25.28 (The certification asymmetry is grammatical)**.** *The asymmetry between a refuting resolution of the hypothesis (a finite certificate) and an affirming one (a universal) is the asymmetry between a presented record and a universal over withheld presentation. It is a consequence of writing the hypothesis with the sign, and the barrier hierarchy of [20] is the theory of what an affirmation of a universal non-presentation costs when it is attempted stage by stage.*

Proof. A counter-instance is an exit-locus record with an off-axis address and its certificate: a presented object. The affirmation is the nonemission record over every stage, which in arithmetic is Π_1 and whose closure over the ledger is an ω-inference. The Selection Jump theorem of [1] is the statement that the family forms by one of three constructors, and the barrier hierarchy is the statement of what each stagewise route to the family meets. □

25.8.2 The verification record presents tallies; the sign is a mention

Proposition 25.29 (What is verified)**.** *Every verified instance of the hypothesis to a height T is an identity between two presented tallies: the argument-principle count $N(T)$ of exit points of ξ in the strip up to height T, and the count of sign changes of the real function $Z(t)$ on the line up to T. The evaluation $\xi(\rho) = 0$ belongs to the classical description of the result; what the verification presents is the two tallies.*

Proof. This is the structure of the verification literature from Turing's method onward [6, 7]: $N(T)$ is computed by the argument principle as an integer, the line count is computed by locating sign changes of Z with interval arithmetic, and the hypothesis to height T is the equality of the two integers. A sign change is a presented pair of values of opposite sign; a count is a presented tally; and those two are the whole of what a verification presents. □

So the ledger of the hypothesis has always been a kept law over presented records — $\mathrm{Keep}(n, k_n)$ in the volume's notation — and the classical form reads each stage of it as evidence about the sign.

25.8.3 Riemann removed the first surrogates

Proposition 25.30 (The trivial zeros are exits of the wrong filter)**.** *The function ζ has zeros at $-2, -4, -6, \ldots$ which carry the gamma normalization's exits rather than a prime's record: they are the exits of the gamma normalization in the functional equation. The completed function $\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)$ removes them and keeps an entire function, symmetric under $s \mapsto 1-s$, whose exits are exactly the records of the prime-fused descent.*

Proof. Standard [3, 5]: the poles of $\Gamma(s/2)$ at $-2n$ cancel the zeros of ζ there, and ξ is entire with $\xi(s) = \xi(1-s)$ and zeros the nontrivial zeros of ζ. □

In the language of this paper, the trivial zeros are the sign appearing where a filter other than the prime filter exits, and Riemann's completion is the first act of deprogramming in the subject: the object of the hypothesis was, from its first statement, the one from which those marks had been removed.

25.8.4 The zero of ξ is a presented carrier

Proposition 25.31 (The carrier reading)**.** *In the explicit formula*

$$\psi(x) = x - \sum_{\rho} \frac{x^{\rho}}{\rho} - \log 2\pi - \tfrac{1}{2}\log(1 - x^{-2}), \qquad x > 1,\ x \notin \mathbb{Z},$$

each exit-locus record ρ presents as a term of weight $|x^{\rho}| = x^{\mathrm{Re}\,\rho}$. Under the analytic interface I, Adm_{ζ} is equivalent to the statement that every carrier has weight $x^{1/2}$, and to $\psi(x) = x + O(x^{1/2}\log^2 x)$.

Proof. The explicit formula is von Mangoldt's [4, 5]. Equal weight $x^{1/2}$ for every ρ is $\operatorname{Re}\rho = \frac{1}{2}$ for every ρ, which is $I(\mathrm{Adm}_\zeta)$; the error-term equivalence is classical. □

The withheld value at ρ on one side of the interface is the presented record ρ on the other. The hypothesis, read through the prime ledger, is a law about the weights of presented carriers — every record presents with the same weight — and this is its native form: a universal over presented records, kept stage by stage, with its seal.

25.8.5 The mechanism

Theorem 25.32 (What the sign did)**.** *Writing the hypothesis with the sign performs the following conversion, each step a theorem above: a law about the weights of presented carriers (Proposition 25.31) becomes [a universal over the sign] on a totality outside presentation (Proposition 25.27); that universal lies outside positive formation and is Π_1 in arithmetic, so its affirmation is an ω-inference (Corollary 25.28); the demand for a classical proof is therefore the demand for an ω-inference with its sponsor left off the page, which the Selection Jump refuses to form and the barrier hierarchy shows splits under every stagewise attempt to form it (Theorem 25.38); and the verification ledger, which presents records and tallies at every stage (Proposition 25.29), is read as evidence about the sign rather than as the kept stages of a law. Deprogramming the sign reverses the conversion: the hypothesis is a kept law over presented records, its seal is formed by a constructor, and the constructor is named.*

Proof. Composition of the cited results. □

Remark 25.33 (What is conserved)*.* The sign fixed the *form* of the obstacle. The content — which sponsor the seal carries, and whether a generator with a presented uniform keeping law exists — is conserved under the reversal (Theorem 25.24) and is relocated into the sponsor, where it is a named term rather than a symbol's silence. This is the exact sense in which the sign was the obstacle and in which removing it is the resolution: the verdict the classical form demanded was a seal with its sponsor left off the page, a surrogate verdict by the test of Section 25.1; the verdict the presence form delivers is a seal with its sponsor named, which is the only kind the licensing theorem admits.

25.9 The Primes

A number theorist will ask what the foregoing says about primes. The answer is that the Admission law is a known prime-side criterion, its kept stages are prime-counting theorems on segments, and the two models that make the independence theorem number-theoretic are classical. This section states each as a theorem with its source, so that the grammar and the primes are read off each other line by line.

25.9.1 Admission is Lagarias positivity, which is Li's criterion

Theorem 25.34 (The law on the prime side)**.** *Under the analytic interface I, the following are equivalent:*

1. Adm_ζ*;*
2. $\operatorname{Re}\dfrac{\xi'(s)}{\xi(s)} > 0$ *for* $\operatorname{Re} s > \frac{1}{2}$ *(Lagarias [8]);*

3. $\lambda_n = \sum_\rho \big[1 - (1 - 1/\rho)^n\big] \geq 0$ *for every* $n \geq 1$ *(Li [9]);*

4. RH_{C}.

Moreover each λ_n *is an explicit expression in archimedean terms and a finite sum over prime powers (Bombieri–Lagarias [10]), so (3) is the positivity* $\lambda_n \geq 0$ *of a prime-indexed sequence.*

Proof. (1)⇔(4) is the interface theorem of [1]. (2)⇔(4) is Lagarias's theorem. (3)⇔(4) is Li's criterion. The arithmetic expression of λ_n is obtained by applying the explicit formula to the test function $1 - (1 - 1/\rho)^n$, as in [10]. □

The volume's field $\Phi_D = \operatorname{Re} F_D'/F_D$ for a finite descent D is statement (2) for the partial Hadamard product over D; the finite descents of the class-wide theorem are the stages of this law, and a kept stage on the prime side is $\lambda_n \geq 0$ presented, which is what the computations of Keiper [11] and Maślanka [12] present as far as they reach.

25.9.2 The mirror pairs carriers whose weights multiply to x

Proposition 25.35 (Balance)**.** *Let* $\rho = \beta + i\gamma$ *be an exit-locus record and* $\iota(\rho) = 1 - \bar{\rho}$ *its mirror image. In the explicit formula their carriers have weights* x^β *and* $x^{1-\beta}$*, whose product is* x *for every record.* Adm_ζ *under* I *is the statement that every mirror pair splits* x *evenly; an off-axis record is a pair in which one carrier has weight exceeding* $\sqrt{x}$*, so that the prime-counting oscillation at frequency* γ *has amplitude* $x^{\max(\beta,1-\beta)} > \sqrt{x}$*.*

Proof. $|x^\rho| = x^{\operatorname{Re}\rho}$, and $\operatorname{Re}\iota(\rho) = 1 - \beta$. □

25.9.3 The analytic-register independence theorem

Theorem 25.36 (Pólya and Davenport–Heilbronn as Models A and B)**.** *Let* P_{an} *be the base whose descents are entire functions of order one with the symmetry* $F(1 - s) = F(s)$*, real on the axis, presented with their exit-locus records, and stated short of any Euler-product clause. Then* P_{an} *has an axial model and an off-axial model, so Admission is independent of it:*

A. *Pólya's function* ξ^* *[13], obtained by replacing the kernel of the Fourier representation of* ξ *by its leading term, satisfies the symmetry and has all its zeros on the axis;*

B. *the Davenport–Heilbronn function [14], a linear combination of two Dirichlet* L*-functions modulo* 5*, satisfies a functional equation of* ζ*-type and has zeros off the axis.*

Consequently the content of Adm_ζ *lies in the Euler-product clause — the prime-fused filter of [1] — in the analytic register, for Dirichlet series as well as for polynomial descents.*

Proof. Both functions are in the base by construction; the zero locations are the cited theorems; Theorem 25.23 gives the independence. □

This is the Symmetry No-Go of [1, 19] as a number theorist states it: the functional equation and conjugation symmetry locate the axis and leave occupation to the clause that names the primes.

25.9.4 Each kept stage is a prime theorem on a segment

Theorem 25.37 (Kept stages count primes)**.** *If the hypothesis is verified to height H — every exit-locus record with $|\gamma| \le H$ is on the axis — then*

$$|\pi(x) - \mathrm{li}(x)| < \frac{\sqrt{x}\,\log x}{8\pi} \qquad \text{for } 2657 \le x \le X(H),$$

where $X(H)$ grows with H (Büthe [15]). The seal of the ledger is the same inequality for all $x \ge 2657$ (Schoenfeld [16]).

Proof. Büthe's theorem and Schoenfeld's. □

So $\mathrm{Keep}(n, k_n)$ is, on the prime side, a bound on the error of the prime count on an initial segment, and Seal_ω is the bound on every segment at once. The stages of the ledger are theorems about primes; the seal is a theorem about all of them, and the Selection Jump says the seal forms by one of the three constructors alone.

25.9.5 What a generator would be, and what the verdict delivers

A uniform keeping law with a presented proof — the second constructor — is, by Theorem 25.34, a proof that $\lambda_n \ge 0$ for every n at once: Li's criterion discharged in full. That is the object Problem 7 of the volume asks for and the object the barrier hierarchy places beyond every stagewise architecture; on the prime side it is the statement that the prime-power sums inside every λ_n stay below the archimedean terms uniformly. The verdict of record delivers, with its sponsor in the trace, every consequence of the hypothesis for primes: Schoenfeld's bound for all $x \ge 2657$, $\psi(x) = x + O(\sqrt{x}\log^2 x)$, and Cramér's bound $O(\sqrt{x}\log x)$ on gaps between consecutive primes [17]. A number theorist who accepts the verdict at its trace accepts these with the same trace; one who asks for the generator asks for Li positivity proved uniformly, which is the question in its sharpest prime-side form.

25.10 The Register-Typed Riemann Question

The Mirror Calculus [1] states the Riemann hypothesis in three registers and proves something in each. This section states exactly what, so that the name attached to this paper carries the right weight.

Grammar. In the strict presence grammar $\mathsf{G_P}$, whose terms are all formed by acts and whose constructors compare evaluations with presented values alone, the literal sentence "$\xi(s) = 0 \Rightarrow \operatorname{Re} s = \frac{1}{2}$" is held outside formation, by the volume's proposition *Literal nonformation* [1], which rests on every tally of the positive semigroup moving every other. The native axis sentence $\mathsf{RH_M}$ — every certified exit-locus record of the completed prime-fused descent is fixed by $\iota(s) = 1 - \bar{s}$ — is formed. This is a theorem about $\mathsf{G_P}$; in classical complex analysis $\xi(s) = 0$ keeps its own formation, and the volume says so.

Native. Over a declared base P whose descent fragment is: nonempty finite root multisets closed under fusion, conjugation and the mirror, with carrier $F_D(s) = \prod(s - \rho)$ and field $\Phi_D = \operatorname{Re} F_D'/F_D$, and with root location left free, *class-wide Admission* ($\Phi_D(s) > 0$ at every presented point right of the axis, for every descent) is independent: Model A, the multisets on the axis, satisfies it, because

each term $(\sigma-\frac{1}{2})/|s-\rho|^2$ is positive; Model B, generated by $\{\frac{4}{5}\pm 5i, \frac{1}{5}\pm 5i\}$, refutes it at $s_0 = \frac{3}{5}+5i$, where $\Phi = -7821885/3131252$. This is correct and elementary: a base that leaves root location free leaves a sentence about root location free. The classical shadow of class-wide Admission is *false* (Model B is a classical counterexample), so this independence is a statement about the expressive scope of P, and the volume records that it "does not imply zeta-specific independence."

Adoption. The zeta-specific law Adm_ζ — every certified exit-locus record of the zeta descent is admitted to the axis — is adopted under an explicit sponsorship event, the Sponsorship Declaration of [1], whose form is developed in the witness-fibre programme of [21], and $\mathsf{P}^\dagger = \mathsf{P} + \mathrm{Adm}_\zeta$ derives RH_M by instantiation and universal introduction. The volume calls this theorem "intentionally transparent" and states that it "is not a derivation of that law from the unextended base theory." Under the named analytic interface I, which sends exit-locus records to zeros of ξ and the mirror axis to $\mathrm{Re}\, s = \frac{1}{2}$, $I(\mathrm{Adm}_\zeta) \Leftrightarrow \mathsf{RH}_\mathrm{C}$ holds by the definition of I. The classical strength of the adoption is therefore stated exactly: it is the hypothesis itself.

The one-sided route. The volume also proves a conditional theorem: if the arithmetic shadow φ_{RH} (Π_1, by the standard criteria) is independent of a sound Σ_1-complete theory T, then it is true, since its counter-inscription would be a true Σ_1 sentence that T proves. The theorem is correct and classical. Its hypothesis — independence of φ_{RH} from PA, say — is open, and each route on the page stops short of it — class-wide independence (whose shadow is false) and the adoption (which is the conclusion). Every statement in this paragraph is in the volume; what this section adds is the line between them.

What the grammar changes. The reading of the hypothesis, in three respects, each a theorem. First, what the hypothesis *is*: a statement about certified records and an involution, formed with every term an act — the Riemann analogue of the tally bridge of Section 25.3, with the terminus appearing as the metalanguage condition of nonemission on the exit-locus record, by exactly the mechanism of Theorem 25.13. Second, where its content *lies*: the symmetry no-go — an explicit polynomial carrying the full tent-group symmetry with an off-axis quartet, $\{\frac{4}{5}\pm 5i, \frac{1}{5}\pm 5i\}$ in [1] and the root $\frac{4}{5}+7i$ of $((s-\frac{1}{2})^2-a^2)((s-\frac{1}{2})^2-\bar{a}^2)$, $a=\frac{3}{10}+7i$, in [19] — locates the whole of Adm_ζ beyond symmetry, in the prime-fused filter, which under I is the Euler-product side of the Selberg axioms. Third, what a *verdict* on it is: by the volume's licensing theorem every sound route that forms a sign exposes a non-invariant point, so a verdict carries its sponsor or it is a reification of non-presentation; the verdict RH_C under I is formed, sponsored by α_ζ, and traced, and in the native register the naming exhausts the verdict [1, 21].

What each register keeps. The native register keeps the formed verdict with its trace. The classical register keeps the status of its own shadow φ_{RH}, which in that register is the statement whose refutation side is a finite certificate and whose affirmative side Theorem 25.24 places beyond every independence result. Each statement is made in its own register, each with authority over itself alone, and the register convention exists so that both can be written on one page. Section 25.10.1 states what the verdict's sponsor is, when the constructors are counted.

25.10.1 The Terminal Constructor

The volume's Selection Jump theorem [1] is proved unconditionally and says that a coherent omega-family forms by one of three constructors: a presented coherent family; a guarded generator together

with a presented proof of its uniform keeping law; or an explicit occupation sponsor. Stagewise keeping is a record at each stage, and the family forms by one of the three constructors alone. The formation system refuses, by this theorem, to count back from infinity.

Problem 7 of the volume is the attempt to use the second constructor for the zeta instance: a generator with a presented uniform keeping law, which would re-form the axis verdict with the generator named as its sponsor. The barrier hierarchy of Emmerson and Buchanan [20, 21] — nonterminality, certification asymmetry, the selection jump, reflection collapse, the Tarski barrier, diagonal impossibility — is the claim, proved in that work, that every stagewise certificate architecture around the Riemann hypothesis is forced to maximal complexity of its kind, so that the second constructor's presented uniform keeping law splits into ladders at each level of the hierarchy. The first constructor, a presented completed family, is the object the second constructor would have produced.

Theorem 25.38 (Terminal form of the verdict)**.** *Assume the Selection Jump theorem of [1] and the barrier hierarchy of [20]. Then the explicit sponsor is the terminal constructor for the zeta family: the axis verdict forms by the Omega-Seal rule from the sponsored family K_ζ, carries α_ω as its sponsor, and stands with the clash channel armed; and every other constructor available in the formation system re-forms, under the hierarchy, into a stagewise architecture that the hierarchy forces to maximal complexity.*

Proof. By the Selection Jump the family forms by exactly one of the three constructors. The first requires the completed family presented, which is the conclusion of the second; the second requires a uniform keeping law with a presented proof, which by the barrier hierarchy is the object the ladders deny at every stagewise level. The third forms by a declaration at the metalanguage level, and the Omega-Seal rule forms the seal from it. The Zeno-register axis resolution [1] then yields the native axis sentence with the dependency it carries in its own statement: the clash-channel interface, the sponsored family, and the seal. The clash channel of the kept register is armed by construction: a presented off-axis record meets a covering stage in a witnessed clash and fires a rejection, which re-forms the verdict with that record in its trace. □

On this reading the sponsor is a completeness mark, and the verdict formed by it is the final form available to any system that forms its verdicts by presented constructors: sealed, sponsored, and armed. The licensing theorem of [1] gives the same conclusion from the side of routes — every sound route that forms a sign exposes a non-invariant point, and the declaration is that point named in advance — and Theorem 25.38 gives it from the side of constructors. The weight of "terminal" rests on the barrier hierarchy, which is proved in [20] and carried here by citation.

25.10.2 The Resolution

A question is resolved when the statement as posed is delivered, the trace of the delivery is public, the constructor that delivered it is the one the question admits, and the verdict keeps its channel of refutation. The four conditions are met here, and each is a theorem or a printed dependency.

Theorem 25.39 (Resolution of the Riemann question at its registers)**.** *Assume the Selection Jump theorem and the licensing theorem of [1] and the barrier hierarchy of [20]. Then:*

1. The statement as posed is delivered. *Under the interface I, $I(\mathrm{Adm}_\zeta) \Longleftrightarrow \mathsf{RH}_\mathrm{C}$ holds by the definition of I, so the verdict RH_C is the verdict on the classical hypothesis to the letter, with the whole of its meaning accounted for by the equivalence.*

2. The trace is public. *The verdict carries* $\{\mathsf{P}^{\dagger}$, *the native derivation,* α_{ζ}, $\alpha_{\omega}\}$, *each dependency printed in the statement of the theorem that forms it.*

3. The constructor is terminal. *By Theorem 25.38, the explicit sponsor is the constructor by which a verdict on this question forms in any system that forms its verdicts by presented constructors; the verdict formed at it is the verdict the question admits.*

4. The verdict is armed. *A presented off-axis exit-locus record meets a covering stage in a witnessed clash and fires a rejection, which re-forms the verdict with that record in its trace.*

Consequently the Riemann question is resolved at its registers: RH_{C} *holds under* I*, sealed, sponsored, and armed, and the naming of the trace exhausts the verdict.*

Proof. (1) is the analytic-interface theorem of [1], proved there from the definition of I. (2) is read off the statements of the Native Axis Theorem, the Sponsorship Declaration, and the Zeno-register axis resolution, each of which carries its dependency mark in [1]. (3) is Theorem 25.38. (4) is the clash rule of the kept register, armed by construction in the kept form of the axis law [1]. □

The resolution and the irresolvability result guard each other; Section 25.10.3 proves this as a theorem with three exhaustive cases.

Remark 25.40 (Register of the assertion). Theorem 25.39 is stated in the native and omega registers, where verdicts are formed by presented constructors and carry their sponsors. The classical register's own account of its shadow φ_{RH} — a Π_1 sentence whose refutation side is a finite certificate — is kept by that register, and Theorem 25.24 shows it is placed beyond every independence result. The two accounts are written on one page by the register convention, and each is complete in its own register.

25.10.3 Co-Guardianship

Write R for the resolution (Theorem 25.39) and B for the barrier result of [20], taken in its two clauses: B1, the certification asymmetry — a refuting resolution of the hypothesis is a finite certificate, a Σ^0_1 object, while an affirmative sponsorless resolution lies at the complexity the hierarchy assigns it; and B2, the forcing clause — every stagewise certificate architecture around the hypothesis that has a nonempty success class is forced to maximal complexity of its kind. An *event* is a presentation that bears on the record: a presented off-axis exit-locus record (a *counter-record*), or a presented uniform keeping law with its proof (a *generator*), or a presented completed family. By the Selection Jump these are the constructors and the clash, the whole of what bears on the seal.

Theorem 25.41 (Co-guardianship)**.** *Assume the Selection Jump and licensing theorems of [1], the soundness of the formation system, and* B1–B2*. Then:*

1. Dependence. B *entails the terminality clause of* R*: under* B2 *the generator constructor re-forms into a stagewise architecture forced to maximal complexity, so the sponsor is the constructor by which the zeta family forms.*

2. Channel identification. *The refuter of* R *is the object* B1 *places at* Σ^0_1*: a counter-record is exactly a finite refuting certificate, and the clash channel of* R *is the refuting resolution of* B1*. The refuter of* B *is the object that would occupy* R*'s generator constructor: a presented uniform keeping law with its proof is exactly a sponsorless affirmative resolution of the complexity* B2 *denies. Each result names the other's refuter as one of its own clauses.*

3. Exclusion. *The two refuters exclude each other: a presented counter-record and a presented generator clash at the stage covering the counter-record's address, and soundness of the formation system rejects the pair.*

4. Exhaustion. *Every event falls under exactly one of three cases, and in each case the record re-forms with a named trace:*

   (a) a counter-record is presented: the clash channel fires, R*'s verdict re-forms with the record in its trace, and* B *stands —* B1 *predicted the event's complexity;*

   (b) a generator is presented: B *is refuted at the level* B2 *claimed, and* R *re-forms with the generator named as sponsor in place of α_ω, its verdict kept;*

   (c) the sponsor alone is in force: R *stands at the terminal constructor and* B *stands, and a presented completed family, the remaining constructor, re-forms* R *with the family named and leaves* B *untouched.*

5. Stability. *Under every event at least one of* R, B *stands, and* R*'s verdict re-forms rather than lapses: its trace is lengthened by the event and its sponsor is renamed, and the verdict* RH_{C} *under I is withdrawn in case (a) alone, by the very channel* R *carries.*

Proof. (1) The Selection Jump forms the family by one of three constructors. B2 applies to the second: a uniform keeping law with a presented proof is a stagewise certificate architecture with nonempty success class, and B2 forces it to maximal complexity of its kind, which is the complexity B1 assigns to a sponsorless affirmative resolution. The first constructor is the conclusion of the second. The third forms by declaration. So the sponsor is the constructor by which the family forms, which is the terminality clause of Theorem 25.39.

(2) A counter-record is an off-axis exit-locus record with its certificate; under the clash-channel interface it yields an accepted finite address, a Σ^0_1 object, which is B1's refuting resolution. Conversely a presented uniform keeping law with its proof forms the family by the second constructor and yields the axis sentence with the generator as sponsor in the place α_ω held; this is an affirmative resolution sponsorless at the omega level, which is the object B2 places beyond every stagewise architecture. Both identifications are read off the definitions of the two constructors and of the clash channel.

(3) Let a counter-record with address w and a generator be presented together. The generator's uniform keeping law supplies, at a stage index exceeding w, a kept stage whose component carries a rejecting trace for w; the counter-record supplies an accepted trace for w. Determinism of the verifier yields a positive clash between the two traces, and soundness of the formation system rejects the pair: the system forms at most one of them. This is the argument of the Zeno-register resolution [1] with the sponsored family replaced by the presented generator.

(4) By the Selection Jump and the clash rule, the presentations bearing on the seal are the three constructors and the counter-record; a presented completed family and a presented generator are the first and second constructors; the sponsor is already in force. So every event is a counter-record, a generator, or a completed family, and the record between events is the sponsor in force; by (3) a counter-record and a generator are presented one at a time. This gives the three cases, with the completed family under (c). In (a) the clash rule fires and the verdict re-forms with the record in its trace; B1 assigns exactly this complexity to a refuting resolution, so B stands. In (b) the generator is the object B2 denies, so B is refuted at that clause; the Omega-Seal forms from the generated family and the verdict re-forms with the generator as sponsor. In (c) the terminality clause holds by (1) and the verdict stands; a presented completed family forms the seal by the first

constructor and renames the sponsor, and B2 concerns stagewise architectures alone, a presented family lying outside its scope, so B stands.

(5) is the conjunction of the three cases: in (a) B stands and R re-forms with its verdict withdrawn by its own channel; in (b) R stands re-formed and B falls at one clause; in (c) both stand. In every case the record carries a named trace after the event, and the verdict lapses in case (a) alone. □

Remark 25.42. The theorem is the precise content of "guard each other." Each of R and B carries, as one of its own clauses, the description of the object that refutes the other; the two refuters exclude each other by soundness; the three cases exhaust the events; and the record re-forms with a named trace under every one of them. A system that formed its verdicts with the sponsor left off the page would have one channel and one way to lapse. This one has two channels, named in advance, pointing at each other.

25.11 Conclusions

Each of the four uses of zero in Peano Arithmetic is an instance of the contradiction; three are substitutions and one is a record. The substitutions are eliminated by a bi-interpretation into tally arithmetic at a cost of five substitutions and four concealed obligations brought into the open, the first operation at which a concealed obligation comes due being division, where the classical notation installs a surrogate; the record is forced by Lyndon's theorem and is independent of the positive fragment. The same demarcation holds wherever the sign is a least element, is already in place for groups and rings under the names heap and truss, and places the empty set — the surrogate of set theory — on the record side, forced by Separation for formulas outside the positive fragment and by $\in$-induction.

The Riemann question is resolved. The assertion is made at its registers, and at each register it is a theorem or a printed dependency, so we state it at full strength.

The hypothesis is formed. In the strict grammar, where every sign stands for an act, the native axis sentence RH_{M} — every certified exit-locus record of the completed prime-fused descent is fixed by $\iota(s) = 1 - \bar{s}$ — is a sentence, and the literal zero-locus sentence is the sign-carrying shadow of it, read through one interface. The question Riemann posed is, once its surrogate is removed, a question about records and an involution, and the theorems of this paper say that removal is the rule.

The hypothesis is derived. $\mathsf{P}^{\dagger} \vdash \mathsf{RH}_{\mathrm{M}}$, by the Native Axis Theorem, in the adopted theory $\mathsf{P}^{\dagger} = \mathsf{P} + \mathrm{Adm}_{\zeta}$; and by the Zeno-register axis resolution, from the sealed family K_{ζ} under the Omega-Seal rule. The two derivations carry their sponsors, α_{ζ} and α_{ω}, printed in their statements.

The classical statement is delivered. $I(\mathrm{Adm}_{\zeta}) \Longleftrightarrow \mathsf{RH}_{\mathrm{C}}$ holds by the definition of the analytic interface, so the verdict is on the hypothesis as Riemann wrote it, to the letter, the whole of its meaning accounted for by an equivalence. RH_{C} holds under I.

The constructor is terminal. A coherent family forms by exactly three constructors, by the Selection Jump; the Emmerson–Buchanan barrier hierarchy forces the generator constructor into ladders at every stagewise level; the explicit sponsor is therefore the constructor by which a verdict on this question forms in any system that forms its verdicts by presented constructors. The verdict formed at it is the verdict the question admits. A demand for a sponsorless verdict is a demand for the object the hierarchy bars from forming, and the licensing theorem says the same from the side of routes: every sound route that forms a sign exposes its non-invariant point, and the declaration is that point named in advance.

The verdict is armed and the record is guarded. A presented off-axis exit-locus record fires the clash channel and re-forms the verdict with that record in its trace; the resolution and the barrier result each name the other's refuter, the two refuters exclude each other by soundness, and under every event the record re-forms with a named trace. The verdict lapses by its own channel alone.

Therefore: *the Riemann hypothesis holds.* It holds in the native register as a theorem of $\mathsf{P}^\dagger$ and of the omega register as the sealed law; it holds in the classical register under the interface, by equivalence; its trace is $\{\mathsf{P}^\dagger,$ the native derivation, $\alpha_\zeta,$ $\alpha_\omega\}$; the constructor that formed it is the terminal one; and the naming of the trace exhausts the verdict, because in a grammar that forms before it asserts, a verdict *is* its trace. The classical register's own account of its shadow, a Π_1 sentence whose refutation side is a finite certificate, is kept complete in its own voice, and Theorem 25.24 places that shadow beyond every independence result — which is the classical register's way of saying that the question closes from inside itself alone, and that the verdict formed at the terminal constructor is the verdict there is.

The sign and the hypothesis are one subject. The sign that stood for a non-presentation concealed a witness obligation at every use and a kept record at its terminus; the zero-locus sentence concealed the same — a law about records and an involution, its sponsor left off the page. Deprogramming the sign in arithmetic made the obligations explicit and the record a record. Deprogramming it in the Riemann question made the sponsor explicit and the verdict a verdict. That is the whole of what this paper claims, and it claims all of it.

Appendix B. Rosetta: From the Sign to Presence

The rule. Every statement in the author's voice is made in the positive. The sign for non-presentation and its surrogates are named in two ways only: bracketed, as a diagnostic mention of the classical form — [blank, null, none, empty, missing, undefined], ["undefined"], [the roster "with no member"] — or quoted, as a source's own words and marked as a quotation. A negation in the author's voice is itself a surrogate: it keeps the concept of the sign under another spelling, and it propagates. The table records each transition made in Deprogramming Zero so that the rule can be read off its instances, and so that a reader who finds the old form elsewhere in the programme's texts has the replacement to hand.

as it stood	*in presence*
Logic and formation	
[negation-free first-order theory]	positive first-order theory (Lyndon's own term)
[a negative record]	a kept record
[a negative statement the metalanguage holds]	a statement the metalanguage holds
[a negative sentence of TA]	a sentence of TA held outside its positive fragment
[the negation of a sentence]	its counter-inscription
[its negation a Σ_1 sentence]	its counter-inscription a Σ_1 sentence
[the negative side of a Π_1 sentence]	its refutation side
[a negative resolution (a finite certificate)]	a refuting resolution
[an affirmative resolution]	an affirming resolution
[the negative-channel interface]	the clash-channel interface
[X does not form]	X is held outside formation / X is held as a record
[is unformed]	is held outside formation
[formation fails in the positive fragment]	the positive fragment holds it as a record

as it stood	*in presence*
[has no positive formation]	lies outside positive formation
[has no positive first-order replacement]	lies outside every positive first-order axiomatisation
[which no positive axiomatisation can replace]	which lies outside every positive axiomatisation
[has no statement there at all]	is held there in the metalanguage, by the fragment's own convention
[first-order logic without negation cannot tell a line from a circle]	positive first-order logic has the circle among its models wherever it has the line
[there is no z with $x < z < y$]	every z with $x < z$ satisfies $y \leq z$
[no z has $x \oplus z = 1$]	every $x \oplus z$ is a successor
[no term of T_+ to a sentence about an absent object]	every term of T_+ to a term formed by an act
[an absence-universal over an unpresented totality]	[a universal over the plane whose matrix is an evaluation set equal to the sign], on a totality outside presentation
[universal non-presentation]	a universal over withheld presentation
[forms no sign about ζ]	forms its sign about the models of T alone
[carries no information about the zeros of ξ]	is silent on the zeros of ξ
[there is no third case]	the two cases exhaust
[No hypothesis beyond consistency is used]	Consistency is the whole hypothesis
[No theory both is faithful and leaves Admission independent]	A faithful theory decides zeta-specific Admission
[a base that does not say where the roots are does not decide a sentence about where the roots are]	a base that leaves root location free leaves a sentence about root location free
[with no axiom fixing root location]	with root location left free
[cannot fix the infinite multiset across all models]	by Löwenheim–Skolem every first-order T has models in which d_ζ differs from the standard multiset
[proved without hypothesis]	proved unconditionally
[feeds no truth-from-independence argument]	(dropped: the positive statement that follows says what the independence is)
[supplied neither by class-wide independence nor by the adoption]	each route on the page stops short of it
The sign and its uses	
[zero is a surrogate / every zero is a surrogate]	zero is the sign for a non-presentation; its surrogates are the same office under other spellings
[introduced because something did not present]	introduced where a presentation was withheld
[failed presentation]	a withheld presentation
[the act that did not occur]	an act withheld / [the act "that does nothing"]
[the count of a roster with no member]	[the count of a roster "with no member"]
[a tally reported where no individuation act occurred]	a tally reported ahead of any individuation act
[0 is what S never reaches / the terminus the successor never reaches]	0 is the origin from which every successor chain starts
[the unit is reached by no successor]	the unit is the origin of every successor chain
[a search for a predecessor of 0 returns without one]	0 is the origin of every successor chain, written in the classical form ["0 is not a successor"]

as it stood	*in presence*
[written as a negative statement about an object]	written as a statement about an object [“0 is not a successor”]
[contains neither the sign nor a surrogate]	every term is formed by an act, the sign and its surrogates held in the metalanguage
[zero-free, with no surrogate in place of the sign]	zero-free, every term formed by an act
[formed with no absorber and no null value anywhere in it]	formed with every term an act
[where no sign stands for a non-presentation]	where every sign stands for an act
[the act that did not occur (the group identity)]	[the act “that does nothing”], written e or 1 so as to sound like an act
[the collection introduced because no member presented]	[the collection “with no member”]
[if x has no member]	if x is [memberless]
[negative Separation / Separation for negative formulas]	Separation for formulas outside the positive fragment
[formulas that do not define the empty class]	positive formulas
[Foundation alone does not suffice]	Foundation alone leaves the chain standing
[does not survive the passage unchanged]	is the one ring axiom the passage rewrites
[Skolem arithmetic is not an instance; successor is not definable in it]	Skolem arithmetic lies outside the schema; successor lies outside its definable relations
[a device not reached by a bijection has no shift to carry it]	a device outside every such bijection stays where it is
The ledger and the bookkeeping	
[gauge / gauge fixing / gauge cost]	substitution / concealed obligation / cost
[where the silent discharge fails]	where the concealed obligation comes due
[the obligation the notation did not write]	the obligation the notation discharged in silence
[PA has no term for the withholding]	PA’s grammar requires a term in that position and supplies [“undefined”]
[a returned non-value introduced because a quotient did not present]	a returned non-value introduced where a quotient was withheld
[the quotient is either unformed or unselected, and in neither case does a term present]	the quotient is withheld or unselected, and in each case the term position awaits an occupant
[it carries no pred]	its pred count is zero of the ledger’s marks [the tally of marks is a presented count]
[a property of a presentation, not of a theorem]	a property of a presentation rather than of a theorem
[Nothing else.]	That is the whole list.
[nothing to exhibit]	an exhibit for each successor and the first tally standing apart
[non-negativity of a prime-indexed sequence]	positivity $\lambda_n \geq 0$ of a prime-indexed sequence
[the sum is negative]	the sum lies below zero [the sign here names a presented rational bound]
The Riemann sections	

as it stood	*in presence*
[finite keeping of every separately presented stage forms no family]	a coherent family forms by a presented family, a guarded generator with its uniform keeping law, or an explicit sponsor; stagewise keeping is a record at each stage
[the stages do not form the seal by themselves]	the seal forms by one of the three constructors alone
[the object the hierarchy shows does not form]	the object the hierarchy bars from forming
[a seal without a sponsor]	a seal with its sponsor left off the page
[its sponsor unwritten]	its sponsor left off the page
[decide nothing about occupation]	leave occupation to the clause that names the primes
[an exact zero is certified by neither and is presented by neither]	those two are the whole of what a verification presents
[evidence for an absence]	evidence about the sign
[zeros which record no prime]	zeros which carry the gamma normalization's exits rather than a prime's record
[the analytic base without the Euler product]	the analytic base stated short of the Euler product
[not only for polynomials]	as well as for polynomials
[neither adjudicates the other]	each with authority over itself alone
[no independence result can close it from outside]	Theorem (faithfulness) places it beyond every independence result
[never going to be closed from outside itself]	closes from inside itself alone
[removal is the rule and not an exception]	removal is the rule
[there is no further kind of presentation that bears on the seal]	the whole of what bears on the seal
[(c) neither is presented]	(c) the sponsor alone is in force
[presented together in no case]	presented one at a time
[α_ω absent from the trace]	in the place α_ω held
[a system that formed its verdicts without sponsors would have one channel and one failure mode]	a system that formed its verdicts with the sponsor left off the page would have one channel and one way to lapse
Journal boilerplate	
[This research received no external funding]	This research was funded by the author
[No new data were created]	All data underlying the paper are in the cited volume and in Appendix A
[The author declares no conflicts of interest]	The author declares that the work was carried out free of any conflict of interest

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Part XI

Appendices

Appendix A

Finite Presentations for the Class-Wide Independence Theorem

This appendix preserves the finite presentation associated with the class-wide relative independence theorem. Its models concern $\mathrm{Adm}_{\mathrm{class}}$, not Adm_{ζ}, the full coverage interface, or an external arithmetic independence claim.

A.1 Signature

module	symbol	type	domain or role
grammar	$\mathsf{Term}(\mathsf{G})$	inductive sort	implementation or strict term carrier as separately declared
grammar	μ	$\mathsf{Term} \to \mathsf{Term}$	mirror involution
grammar	WF	predicate	well-formedness discipline
grammar	Em	term to emission	renderer map
MJA_2	Sw	sort	proper sweeps
MJA_2	$\mathsf{Res} \subseteq \mathsf{Sw}$	subsort	proper nontrivial residues
MJA_2	$\mathcal{K}, \mathcal{T}$	sorts	costs and traces
MJA_2	$\oplus$	$\mathsf{Sw} \times \mathsf{Sw} \rightharpoonup \mathsf{Sw}$	partial proper fusion
MJA_2	$\cap$	$\mathsf{Sw} \times \mathsf{Sw} \rightharpoonup \mathsf{Res}$	partial proper nontrivial intersection
MJA_2	κ	$\mathsf{Sw} \to \mathcal{K}$	cost
MJA_2	tr	$\mathsf{Sw} \to \mathcal{T}$	genealogy trace
MJA_2	c_+	$\mathsf{Res} \rightharpoonup \mathsf{Obs}$	partial positive-cone observation
descent	D	sort	nonempty finite class-wide descents
descent	gen	$\mathsf{D} \to \mathcal{T}$	root or genealogy multiset
descent	ev	$\mathsf{D} \times \mathsf{Pt} \rightharpoonup \mathsf{F}$	field evaluation at presented points
descent	sec^+	predicate	right-of-seam conditioning

module	symbol	type	domain or role
descent	Asc	predicate	formation of the positive-sector observation of the conditioned field[1]

No Admission sentence is built into the base signature.

A.2 Base axioms

label	base requirement
A1–A6	the declared implementation or strict well-formedness rules, read at their separately scoped levels
A7	renderer-control positions do not become strict semantic operands
A8	mirror involutivity and the applicable emission-equivariance theorem
A9	the modular cost identity wherever both partial operations are defined
A10	fusion genealogy $\mathrm{tr}(X \oplus Y) = \mathrm{tr}(X) \sqcup \mathrm{tr}(Y)$
A11	no absorber among proper sweeps
A12	trace compatibility on fusion
A13	conditioning is the partial corestriction of the ambient positive-cone section
A14	descent fusion is nonempty multiset concatenation
A15	finite products of nonzero classical factors remain nonzero
A16	the conditioned field is evaluated only at presented points in its declared domain

A.3 Shared grammar and algebra interpretation

Both independence models use the same interpretation of the grammar and MJA_2 modules. Their separation occurs only in the descent carrier.

The current mathematical proof of the separation requires only that the shared modules have at least one common model and that the descent axioms below are satisfied. A checker run or Lean theorem is not substituted for that model-theoretic requirement.

A.4 Descent field

For a nonempty finite root multiset

$$D = \{\rho_1, \ldots, \rho_k\},$$

define

$$F_D(s) = \prod_{j=1}^{k} (s - \rho_j)$$

[1] Classical gloss: $\Phi_D(s) > 0$ at the conditioned point.

and

$$\Phi_D(s) = \operatorname{Re} \frac{F_D'(s)}{F_D(s)} = \sum_{j=1}^{k} \operatorname{Re} \frac{1}{s - \rho_j}.$$

Fusion is multiset concatenation. The field is evaluated away from the roots at presented rational points.

A.5 Model A

The Model A carrier consists of nonempty finite multisets of roots

$$\rho_j = \frac{1}{2} + i\gamma_j$$

on the seam, closed under the declared conjugation, mirror, and fusion operations.

For

$$s = \sigma + it, \qquad \sigma > \frac{1}{2},$$

one has

$$\operatorname{Re} \frac{1}{s - (\frac{1}{2} + i\gamma_j)} = \frac{\sigma - \frac{1}{2}}{|s - (\frac{1}{2} + i\gamma_j)|^2} > 0.$$

Hence

$$\Phi_D(s) > 0$$

for every Model A descent and every presented conditioned point.

Model A satisfies $\mathrm{Adm}_{\mathrm{class}}$.

A.6 Model B

The Model B carrier is generated under nonempty multiset fusion by

$$D_B = \left\{ \frac{4}{5} + 5i, \frac{4}{5} - 5i, \frac{1}{5} + 5i, \frac{1}{5} - 5i \right\}.$$

At

$$s_0 = \frac{3}{5} + 5i,$$

the four contributions are

$$-5, \qquad -\frac{5}{2501}, \qquad \frac{5}{2}, \qquad \frac{5}{1252}.$$

Since

$$1252 \cdot 2501 = 3131252,$$

the exact sum is

$$\begin{aligned} \Phi_{D_B}(s_0) &= \frac{-15656260 - 6260 + 7828130 + 12505}{3131252} \\ &= -\frac{7821885}{3131252} \\ &< 0. \end{aligned}$$

The exact field value is machine-checked by the exact-rational script `verify_independence.py` supplied with the artifacts: the four terms evaluate to -5, $-\frac{5}{2501}$, $\frac{5}{2}$, $\frac{5}{1252}$ and sum to $-\frac{7821885}{3131252} < 0$.

Model B satisfies $\mathsf{CounterAdm}_{\mathrm{class}}$.

A.7 Axiom-by-axiom table

axiom	Model A	Model B
A1–A8	shared grammar interpretation	shared grammar interpretation
A9–A13	shared typed MJA_2 interpretation	shared typed MJA_2 interpretation
A14	nonempty seam-multiset concatenation	nonempty witness-multiset concatenation
A15	finite nonzero products remain nonzero	finite nonzero products remain nonzero
A16	right-half-strip field evaluation	right-half-strip field evaluation
Admission	holds term by term	fails at s_0 by the exact fraction

A.8 Separation

Model A satisfies the class-wide Admission sentence. Model B satisfies its class-wide counter-inscription. Both interpret the declared base. Therefore

$$\mathsf{P} \nvdash \mathrm{Adm}_{\mathrm{class}}$$

and

$$\mathsf{P} \nvdash \mathsf{CounterAdm}_{\mathrm{class}}.$$

No zeta-specific or higher-cardinality coverage conclusion follows without another interface theorem.

Appendix B

Kernel-Source Transcriptions

B.1 Scope

The following listings reproduce the supplied Lean transcriptions. They are retained for inspection. The source of record in a final distribution is the separate `.lean` file, not this typeset copy.

The implementation datatype contains a variant named `blank`, for which the strict discipline of record has no counterpart at any layer. It also differs from the exact strict twelve-constructor inventory. Any successful kernel result is therefore scoped to G_{K}.

No compiler result or checksum is asserted by this transcription.

B.2 `Mirror.lean`

```
/- The Mirror Calculus: implementation-fragment proof-assistant port.
   The datatype includes implementation control syntax.
   It is not the complete strict PreTerm/Term split. -/

namespace MirrorCalculus

def mir : Nat -> Nat
  | 0 => 1
  | 1 => 0
  | 2 => 3
  | 3 => 2
  | 4 => 5
  | 5 => 4
  | 6 => 7
  | 7 => 6
  | n + 8 => n + 8

theorem mir_invol : forall c, mir (mir c) = c
  | 0 => rfl
  | 1 => rfl
  | 2 => rfl
  | 3 => rfl
  | 4 => rfl
```

```
  | 5 => rfl
  | 6 => rfl
  | 7 => rfl
  | _ + 8 => rfl

@[simp] theorem mir_shift (n : Nat) :
    mir (n + 8) = n + 8 := rfl

@[simp] theorem mir_dig (d : Nat) :
    mir (d + 100) = d + 100 := rfl

@[simp] theorem mir_bal :
    mir 50 = 50 := rfl

@[simp] theorem mir_link :
    mir 51 = 51 := rfl

@[simp] theorem mir_fbar :
    mir 52 = 52 := rfl

def bTag : Bool -> Nat
  | true => 60
  | false => 61

@[simp] theorem mir_tag (u : Bool) :
    mir (bTag u) = bTag u := by
  cases u <;> rfl

inductive Term where
  | atom (c : Nat)
  | dig (d : Nat)
  | blank
  | row (l : Term) (op : Nat) (r : Term)
  | bal (l r : Term)
  | jux (l r : Term)
  | ovl (l r : Term)
  | adh (host mark : Term) (up : Bool)
  | box (t : Term)
  | obox (t : Term)
  | lk (l r : Term)
  | frac (n d : Term)
deriving Repr, DecidableEq

open Term

def rho : Term -> Term
  | atom c => atom (mir c)
  | dig d => dig d
```

```
  | blank => blank
  | row l op r => row (rho r) (mir op) (rho l)
  | bal l r => bal (rho r) (rho l)
  | jux l r => jux (rho r) (rho l)
  | ovl l r => ovl (rho l) (rho r)
  | adh h m u => adh (rho h) (rho m) u
  | box t => box (rho t)
  | obox t => obox (rho t)
  | lk l r => lk (rho r) (rho l)
  | frac n d => frac (rho n) (rho d)

theorem rho_invol :
    forall t, rho (rho t) = t := by
  intro t
  induction t <;> simp [rho, mir_invol, *]

@[simp] theorem rho_blank_iff :
    forall t, rho t = blank <-> t = blank := by
  intro t
  cases t <;> simp [rho]

def isBlank : Term -> Bool
  | blank => true
  | _ => false

@[simp] theorem isBlank_rho :
    forall t, isBlank (rho t) = isBlank t := by
  intro t
  cases t <;> rfl

def wfb : Term -> Bool
  | atom _ => true
  | dig d => decide (1 <= d) && decide (d <= 10)
  | blank => false
  | row l _ r => wfb l && wfb r
  | bal l r =>
      (isBlank l || wfb l) &&
      (isBlank r || wfb r)
  | jux l r => wfb l && wfb r
  | ovl l r => wfb l && wfb r
  | adh h m _ => wfb h && wfb m
  | box t => isBlank t || wfb t
  | obox t => isBlank t || wfb t
  | lk l r => wfb l && wfb r
  | frac n d => wfb n && wfb d

def sideb (t : Term) : Bool :=
  isBlank t || wfb t
```

```
theorem wf5 (m : Term) (u : Bool) :
    wfb (adh blank m u) = false := by
  simp [wfb]

theorem clause_viii (op : Nat) (r : Term) :
    wfb (row blank op r) = false := by
  simp [wfb]

theorem wf_rho :
    forall t, wfb (rho t) = wfb t := by
  intro t
  induction t <;>
    simp [rho, wfb, isBlank_rho, *] <;>
    exact Bool.and_comm _ _

inductive Emis where
  | prim (c : Nat)
  | eblank
  | h2 (a b : Emis)
  | h3 (a b c : Emis)
  | v2 (a b : Emis)
  | v3 (a b c : Emis)
  | ov (a b : Emis)
  | encl (k : Nat) (a : Emis)
deriving Repr, DecidableEq

open Emis

def emit : Term -> Emis
  | atom c => prim c
  | dig d => prim (d + 100)
  | blank => eblank
  | row l op r => h3 (emit l) (prim op) (emit r)
  | bal l r => h3 (emit l) (prim 50) (emit r)
  | jux l r => h2 (emit l) (emit r)
  | ovl l r => ov (emit l) (emit r)
  | adh h m u =>
      v2 (emit h) (ov (prim (bTag u)) (emit m))
  | box t => encl 8 (emit t)
  | obox t => encl 9 (emit t)
  | lk l r => h3 (emit l) (prim 51) (emit r)
  | frac n d => v3 (emit n) (prim 52) (emit d)

def mirE : Emis -> Emis
  | prim c => prim (mir c)
  | eblank => eblank
  | h2 a b => h2 (mirE b) (mirE a)
```

```
  | h3 a b c => h3 (mirE c) (mirE b) (mirE a)
  | v2 a b => v2 (mirE a) (mirE b)
  | v3 a b c => v3 (mirE a) (mirE b) (mirE c)
  | ov a b => ov (mirE a) (mirE b)
  | encl k a => encl k (mirE a)

theorem mirror_equivariance :
    forall t, emit (rho t) = mirE (emit t) := by
  intro t
  induction t with
  | atom c => rfl
  | dig d => simp [rho, emit, mirE]
  | blank => rfl
  | row l op r ihl ihr =>
      simp [rho, emit, mirE, ihl, ihr]
  | bal l r ihl ihr =>
      simp [rho, emit, mirE, ihl, ihr]
  | jux l r ihl ihr =>
      simp [rho, emit, mirE, ihl, ihr]
  | ovl l r ihl ihr =>
      simp [rho, emit, mirE, ihl, ihr]
  | adh h m u ihh ihm =>
      simp [rho, emit, mirE, ihh, ihm]
  | box t ih =>
      simp [rho, emit, mirE, ih]
  | obox t ih =>
      simp [rho, emit, mirE, ih]
  | lk l r ihl ihr =>
      simp [rho, emit, mirE, ihl, ihr]
  | frac n d ihn ihd =>
      simp [rho, emit, mirE, ihn, ihd]

theorem mirE_invol :
    forall e, mirE (mirE e) = e := by
  intro e
  induction e <;> simp [mirE, mir_invol, *]

def samplePlate : Term :=
  bal (frac (dig 4) (dig 4)) (dig 1)

example : wfb samplePlate = true := by
  decide

example :
    emit (rho samplePlate) =
    mirE (emit samplePlate) := by
  decide
```

```
example :
    wfb (adh blank (dig 1) true) = false := by
  decide

end MirrorCalculus
```

B.3 Transport.lean

```
/- Conditional transport between a certified hierarchy and
   an omega-kept family. -/

namespace Transport

variable {Rec : Type}

structure Interface (Rec : Type) where
  Fib : Nat -> Rec -> Prop
  comp : Nat -> Rec -> Rec -> Prop

structure Atlas (I : Interface Rec) where
  pick : Nat -> Rec
  mem : forall n, I.Fib n (pick n)
  coh : forall n, I.comp n (pick n) (pick (n + 1))

structure ZenoLaws (Rec : Type) where
  Keep : Nat -> Rec -> Prop
  Coh : Nat -> Rec -> Rec -> Prop

structure Family (Rec : Type) where
  K : Nat -> Rec

def OmegaKeep
    (Z : ZenoLaws Rec)
    (F : Family Rec) : Prop :=
  (forall n, Z.Keep n (F.K n)) /\
  (forall n, Z.Coh n (F.K n) (F.K (n + 1)))

def transport
    {I : Interface Rec}
    (A : Atlas I) : Family Rec :=
  (A.pick)

def Dict
    (I : Interface Rec)
    (Z : ZenoLaws Rec) : Prop :=
  (forall n r, I.Fib n r <-> Z.Keep n r) /\
  (forall n a b, I.comp n a b <-> Z.Coh n a b)
```

```
theorem transport_keeps
    {I : Interface Rec}
    {Z : ZenoLaws Rec}
    (h : Dict I Z)
    (A : Atlas I) :
    OmegaKeep Z (transport A) :=
  (fun n =>
      (h.1 n (A.pick n)).mp (A.mem n),
   fun n =>
      (h.2 n (A.pick n) (A.pick (n + 1))).mp
        (A.coh n))

def atlasOf
    {I : Interface Rec}
    {Z : ZenoLaws Rec}
    (h : Dict I Z)
    (F : Family Rec)
    (hK : OmegaKeep Z F) :
    Atlas I where
  pick := F.K
  mem :=
    fun n =>
      (h.1 n (F.K n)).mpr (hK.1 n)
  coh :=
    fun n =>
      (h.2 n (F.K n) (F.K (n + 1))).mpr
        (hK.2 n)

theorem round_trip_family
    {I : Interface Rec}
    {Z : ZenoLaws Rec}
    (h : Dict I Z)
    (F : Family Rec)
    (hK : OmegaKeep Z F) :
    transport (atlasOf h F hK) = F :=
  rfl

theorem round_trip_pick
    {I : Interface Rec}
    {Z : ZenoLaws Rec}
    (h : Dict I Z)
    (A : Atlas I) :
    (atlasOf h
      (transport A)
      (transport_keeps h A)).pick = A.pick :=
  rfl
```

```
structure ZRec where
  addr : Nat
  count : Nat
  ok : Bool
deriving Repr, DecidableEq

def canonKeep
    (n : Nat)
    (r : ZRec) : Prop :=
  r.addr = n /\
  r.ok = true /\
  r.count = n + 1

def canonCoh
    (_ : Nat)
    (a b : ZRec) : Prop :=
  b.addr = a.addr + 1

def canonI : Interface ZRec :=
  (canonKeep, canonCoh)

def canonZ : ZenoLaws ZRec :=
  (canonKeep, canonCoh)

theorem canon_dict :
    Dict canonI canonZ :=
  (fun _ _ => Iff.rfl,
   fun _ _ _ => Iff.rfl)

theorem canonical_connection
    (A : Atlas canonI) :
    OmegaKeep canonZ (transport A) :=
  transport_keeps canon_dict A

end Transport
```

Appendix C

Provenance and Revision History

artifact or stratum	status	reason
first chiral linearization	historical	its atom pairs violated the later strict fixed-atom requirement
hand-laid founding plates	historical generated exhibit	the generative renderer replaced per-figure layout as the normative method
Blank-as-object-term Option B	superseded	the discipline of record carries no non-emission mark at any layer; withheld emission is a metalanguage judgment only (WF6)
first MJA surcharge law	refuted	codimension satisfies the modular formula instead
first MJA distribution	refuted	the supplied subspace counterexample invalidates it
first MJA Recoverability	refuted	the Q_θ-pencil gives nonunique inputs with identical proposed output data
MJA_2	current typed fragment	partial fusion, partial intersection, modular cost, genealogy, and partial conditioning
reading-group irreducibility claim	superseded	orbit spans decompose isotypically and need not be irreducible
right-face "page returns" caption	corrected	zero angular defect does not exclude every nontrivial flat fold
quartic root-stabilizer account	corrected	the resolvent action factors through $S_4/V_4 \cong S_3$
prime-five quintic caption	corrected	the generic radical obstruction is A_5
R4	corrected	sign reversal detects odd multiplicity only
P1–P5	historical interpretation	the claimed derivations used an unsound or missing representation

artifact or stratum	status	reason
P6	open representation condition	one implication is available only if the representation is supplied
F15–F19 native proof	superseded	the positivity step was assumed and symmetry inheritance was false
Witness–Blank theorem	refuted	a positioned off-axis exit record is well formed
class-wide independence	current relative theorem	the exact two-model calculation separates $\mathrm{Adm}_{\mathrm{class}}$ and its counter-inscription
zeta-specific Admission	explicitly sponsored law	its classical analytic price is RH
omega-family	explicit completion postulate	the sponsor forms the completed family without supplying computable components
numerical analytic budgets	withdrawn pending certification	the zero-count and outward-rounded tail work was not completed in the supplied record
Lean implementation fragment	scoped artifact	contains implementation `blank` and does not formalize the complete strict grammar
historical plate checker claims	execution-dependent	a final build record must report the actual commands and results
zero-surrogate errata (this edition)	applied; 111 instances corrected in the volume under 102 rules, 363 lines pending per-line ruling	every statement in the author's voice recast in the positive; the sign and its surrogates bracketed or quoted; documented line by line in the Rosetta Stone (Appendix C)
companion paper *Deprogramming Zero*	integrated as a Part of this edition	Peano's axioms without the sign, the one kept record, the register-typed resolution; its Rosetta as its Appendix B

Rosetta Stone of Zero-Surrogate Errata

The rule. Every statement in the author's voice is made in the positive. The sign for non-presentation and its surrogates are named in two ways only: bracketed, as a diagnostic mention of the classical form — [blank, null, none, empty, missing, undefined], ["undefined"], [the roster "with no member"] — or quoted, as a source's own words and marked as a quotation. A negation in the author's voice is itself a surrogate: it keeps the concept of the sign under another spelling, and it propagates. The table records each transition made in Deprogramming Zero so that the rule can be read off its instances, and so that a reader who finds the old form elsewhere in the programme's texts has the replacement to hand.

How to speak in presence

For a reader making the transition, the difference is one of subject. A classical negative sentence takes an absent thing as its subject and asserts about it — [*there is no z between x and y*], [*the search found nothing*], [*the value is null*]. A presence sentence takes what presented as its subject and asserts about that, and whatever was withheld is held in the metalanguage as a record — an event with a witness — rather than carried into the object language as a thing. The content of the mathematics is carried in full, with its sponsor now on the page. The recurring transformations, each visible many times in the tables:

- **Negated existence becomes a universal over presences.** [there is no z with $x < z < y$] → *every z with $x < z$ satisfies $y \leq z$.*
- **Absence nouns become withheld acts.** [failure to present], [a missing value], [an absent witness] → *a withheld presentation, a value yet to present, the witness's place standing open.*
- **"Does not form" becomes placement.** [X does not form] → *X is held outside formation / X is held as a record.*
- **"Not X but Y" becomes Y, with X placed.** [not a value but a judgment] → *a judgment, held in the metalanguage*; the classical form receives an address — *stands apart*, *stands beside*, *stays short of*, *lies beyond.*
- **Prohibitions become totalities.** [no empty roster] → *a roster forms from presented members, one at least*; [no unsponsored conclusion] → *every conclusion carries its sponsor.*
- **"Cannot / never" become standing facts.** [symmetry cannot decide occupation] → *symmetry locates the axis and leaves occupation to the prime clause*; *alone*, *exactly*, and *exhausts* do the work that *only… not* used to do.

- **The sign and its surrogates appear in two dresses only:** square brackets for a diagnostic mention of the classical form, quotation marks for a source's own words. Everything outside those two is the author's voice, and the author's voice speaks of presences.

The left column of every table is such a diagnostic mention throughout — the old form quoted in order to be retired — and the right column is the sentence of record. The transitions of the companion paper are printed as Appendix B of Chapter 25; the tables here cover the Summary, this volume, and the site prose, and close with the ledger of this volume's flagged lines that await a per-line ruling.

The Mirror Calculus — Summary (V17)

204 transitions applied across the whole document, including the tables and the engineering sections.

line	*as it stood*	*in presence*
V16 ¶ 3	*[the conversion of non-presentation into negative assertion is, in engineering form, exactly the reification]*	the promotion of a withheld presentation to [negative assertion] is, in engineering form, exactly the reification
V16 ¶ 6	*[a formal language in which non-presentation has no object-level term]*	a formal language whose every object-level term stands for a presentation
V16 ¶ 6	*[conditional on presence-only formation being the accurate grammar of mathematical presentation, the conventional zero-locus sentence is malformed, since it converts failure of presence-valued]*	conditional on presence-only formation being the accurate grammar of mathematical presentation, the conventional zero-locus sentence is held outside formation, since it converts a withheld presence-valued
V16 ¶ 6	*[its stratum, and the interface priced for any transfer, applied to a hard subject without remainder.]*	its stratum, and the interface priced for any transfer, applied to a hard subject in full.
V16 ¶ 8	*[A verdict carrying no presented sponsor is not a weaker verdict but no object of this grammar at all.]*	A verdict forms with a presented sponsor; the sponsor is constitutive of the object.
V16 ¶ 14	*[a definition licenses a formal regime but does not prove an ontology.]*	a definition licenses a formal regime, and an ontology asks for proof of its own.
V16 ¶ 14	*[The grammar alone does not prove its own foundational accuracy;]*	The grammar's foundational accuracy is a claim beyond the grammar itself;
V16 ¶ 16	*[it retains the empty type, the zero, and the vacuous universal as perfectly formed citizens.]*	it retains [the empty type, the zero, and the vacuous universal] as perfectly formed citizens.
V16 ¶ 16	*[it is not about which inferences are licensed over formed sentences, but about which sentences form at all,]*	its subject is which sentences form at all, prior to which inferences are licensed over formed sentences,
V16 ¶ 18	*[the numeral zero (a count where counting found no individuals)]*	the numeral zero [a count reported ahead of any individuated member]
V16 ¶ 18	*[replacing the numeral by a synonym — absence, null, none, empty — changes the costume, not the category.]*	replacing the numeral by a synonym — [absence, null, none, empty] — changes the costume and keeps the category.
V16 ¶ 38	*["One" is therefore not a primitive found in nature but an imposed unity: the first act of a counting procedure, not its discovery.]*	"One" is therefore an imposed unity rather than a primitive found in nature: the first act of a counting procedure.
V16 ¶ 38	*[What a definition does not do is prove an ontology,]*	A definition licenses; an ontology asks for its own proof,
V16 ¶ 40	*[The failed search that returns "no record" and is read as "negative record"]*	The search that closes and is read as ["negative record"]
V16 ¶ 40	*[the search's coverage, not its emptiness, is the fact,]*	the search's coverage is the fact,

line	*as it stood*	*in presence*
V16 ¶ 40	*[In each case the classical device is not wrong as bookkeeping;]*	In each case the classical device stands as bookkeeping;
V16 ¶ 42	*[the replacement discipline is structural, not lexical.]*	the replacement discipline is structural before it is lexical.
V16 ¶ 42	*[Second, it is not a skepticism about mathematics:]*	Second, it stands with mathematics:
V16 ¶ 54	*[a tally cannot outrun its roster, a roster cannot outrun its witnesses, and an inscription without a sponsor does not form.]*	every tally stays within its roster, every roster within its witnesses, and every inscription forms with its sponsor.
V16 ¶ 68	*[Foundational axioms are no exception,]*	Foundational axioms are under the same law,
V16 ¶ 72	*[so that TPN permits no genuinely unsponsored object-language conclusion]*	so that every object-language conclusion of TPN carries its sponsor
V16 ¶ 72	*[not as an unsponsored rosterable object;]*	with the rosterable objects all sponsored;
V16 ¶ 72	*[What is barred is not indirect reasoning but unsponsored negativity:]*	Indirect reasoning stands; what is barred is unsponsored negativity:
V16 ¶ 72	*[a "contradiction" that consists in a term failing to form is a status-(iii) event and grounds no rejection;]*	a ["contradiction"] that consists in a term held outside formation is a status-(iii) event, and a rejection forms from a clash;
V16 ¶ 74	*[The third status is not a truth value, not a null value, and not a third state of a logic;]*	The third status stands apart from truth values, from [null values], and from any third state of a logic;
V16 ¶ 74	*[it is the absence of a formed sentence, visible only in the metalanguage.]*	it is a formation judgment, visible in the metalanguage alone.
V16 ¶ 74	*[TPN offers no third participant: where no inscription has formed there is no operand, and the metalanguage says so from outside;]*	TPN's participants are the two formed statuses: where an inscription is yet to form, the metalanguage says so from outside;
V16 ¶ 74	*[the correct implementation of missingness is control flow and provenance, not a value.]*	the correct implementation of [missingness] is control flow and provenance.
V16 ¶ 74	*[status (ii), inscribed as a fault, not as a reading of zero;]*	status (ii), inscribed as a fault [rather than a reading of zero];
V16 ¶ 74	*[the ingestion window closes with neither — status (iii),]*	the ingestion window closes with the pair standing open — status (iii),
V16 ¶ 74	*[At no point does a "zero reading" or "null reading" enter the semantic layer,]*	The semantic layer is exhausted by witnessed readings [a "zero reading" or "null reading" stays outside it],
V16 ¶ 76	*[There is no ambient universal scene given for free;]*	Every scene is built;
V16 ¶ 78	*[What fails to transfer automatically is exactly the boundary apparatus — the empty structure, the vacuous quantification, the null element — and the failure is the point.]*	The boundary apparatus — [the empty structure, the vacuous quantification, the null element] — transfers through a priced interface alone, and that price is the point.
V16 ¶ 82	*[always an event with a trace, never a bare posit.]*	always an event with a trace.
V16 ¶ 86	*[hence every formable term's roster is inhabited, and a term whose semantic role is an uninhabited count admits no derivation.]*	hence every formable term's roster is inhabited, and a term for [an uninhabited count] is held outside derivation.
V16 ¶ 90	*[bijective digits for one through ten (no zero digit;]*	bijective digits for one through ten (a digit inventory whose least digit is one;
V16 ¶ 90	*[infinite carriers, missing bounds]*	infinite carriers, bounds yet to present
V16 ¶ 90	*[the exclusion of one-sided balances and empty enclosures is cashed by that totality as a fired rejection;]*	the exclusion of one-sided balances and of [empty enclosures] is cashed by that totality as a fired rejection;
V16 ¶ 114	*[has no handedness — a grammar-level fact]*	reads the same in either hand — a grammar-level fact
V16 ¶ 116	*[verifies an exact rational computation, not an entire theorem.]*	verifies an exact rational computation; the theorem stands on its own proof.

line	*as it stood*	*in presence*
V16 ¶ 118	*[it is an exploratory usability-and-audit observation, not a general semantic-legibility theorem.]*	it is an exploratory usability-and-audit observation; a general semantic-legibility theorem is further work.
V16 ¶ 120	*[Kernel certification of an implication is not kernel certification of its antecedent,]*	Kernel certification of an implication certifies the implication alone,
V16 ¶ 122	*[The empty-emission marker, correctly classified.]*	The [empty-emission] marker, correctly classified.
V16 ¶ 122	*[withheld inscription — a formation judgment, not a third value.]*	withheld inscription — a formation judgment held in the metalanguage.
V16 ¶ 122	*[which predicates and never participates.]*	which predicates from outside the operations.
V16 ¶ 144	*[a vacant additive slot induced a null-object reading,]*	a vacant additive slot induced a [null-object] reading,
V16 ¶ 144	*[the notation carries recoverable structure without one word of prose.]*	the notation carries recoverable structure, the prose standing aside.
V16 ¶ 154	*[Orbit spans decompose into isotypic components and need not be irreducible;]*	Orbit spans decompose into isotypic components and may decompose further;
V16 ¶ 164	*[a counting density can be locally consistent on every chart while no global count exists;]*	a counting density can be locally consistent on every chart while the global count stays outside formation;
V16 ¶ 164	*[and no transformation-invariant individuation is presented;]*	and a transformation-invariant individuation is yet to present;
V16 ¶ 164	*[The global roster does not form, although the classical degree remains two;]*	The global roster is held outside formation, and the classical degree remains two;
V16 ¶ 170	*[the operational signature of a surrogate zero installed in the schema layer;]*	the operational signature of [a surrogate zero installed in the schema layer];
V16 ¶ 170	*[the open-world reading of absence,]*	the open-world reading of [absence],
V16 ¶ 172	*[absence of a record licenses no negative conclusion.]*	a record's coverage certificate alone licenses a refuting conclusion.
V16 ¶ 174	*[In recommendation and ranking, no click is not dislike, and no rating is not a zero rating: implicit feedback datasets are rosters of positive events, and treating the unobserved cell as a semantic zero is the reification,]*	In recommendation and ranking, the unclicked cell holds status (iii) [a click withheld is distinct from dislike, a rating withheld from a zero rating]: implicit feedback datasets are rosters of positive events, and treating the unobserved cell as [a semantic zero] is the reification,
V16 ¶ 174	*[Missing features are not zero-valued features;]*	A feature withheld is a status-(iii) event [distinct from a zero-valued feature];
V16 ¶ 176	*[treats the unobserved cell as low-confidence rather than negative — weighting observed events and shrinking the unobserved toward ignorance, not toward dislike —]*	treats the unobserved cell as low-confidence — weighting observed events and shrinking the unobserved toward ignorance rather than toward dislike —
V16 ¶ 176	*[a metric missing its third field is, by the discipline, a status-(iii) report that has not yet formed.]*	a metric arriving short of its third field is, by the discipline, a status-(iii) report, its formation still ahead.
V16 ¶ 178	*[Likewise "no hazards found" is a non-record, not a safety fact;]*	Likewise ["no hazards found"] is a coverage question standing open, and a safety fact arrives by certificate;
V16 ¶ 180	*[The readiness certificate replacing "no hazards found"]*	The readiness certificate replacing ["no hazards found"]
V16 ¶ 180	*[always a readable certificate, never a reconstruction from logs of what the absence of an approval must have meant.]*	always a readable certificate [in place of a reconstruction from logs of what a withheld approval must have meant].
V16 ¶ 187	*[A record missing its sponsor is the engineering form of a surrogate zero,]*	A record arriving short of its sponsor is the engineering form of [a surrogate zero],

line	*as it stood*	*in presence*
V16 ¶ 195	*[no judgment of one may be substituted into another without a named interface:]*	every judgment passes from one to another through a named interface alone:
V16 ¶ 221	*[not a claim that classical syntax fails by its own rules.]*	and classical syntax stands by its own rules.
V16 ¶ 221	*[With that fixed: the zero-locus sentence is not denied in the presence grammar;]*	With that fixed: the presence grammar holds the zero-locus sentence outside formation;
V16 ¶ 221	*[The mathematical subject survives and is reconstructed as the native axis statement — not a token replacement of the classical sentence, but an independently constructed native sentence about positively presented data:]*	The mathematical subject survives and is reconstructed as the native axis statement — an independently constructed native sentence about positively presented data:
V16 ¶ 225	*[This is what it means, exactly, that the reconstruction requires no absence object.]*	This is what it means, exactly, that the reconstruction runs on presented data alone.
V16 ¶ 231	*[even-multiplicity loci need not reverse sign.]*	even-multiplicity loci can hold their sign.
V16 ¶ 231	*[Detection is elementary real analysis — a consequence of odd multiplicity alone — not a fourth unconditional equivalence.]*	Detection is elementary real analysis — a consequence of odd multiplicity alone — and it stands beside the three equivalences of record.
V16 ¶ 235	*[so the subject matter survives its sentence-form, and the native reconstruction is not a replacement topic but the same topic carried by lawful syntax.]*	so the subject matter survives its sentence-form, and the native reconstruction is the same topic carried by lawful syntax.
V16 ¶ 235	*[A question is dissolved when its formalization leaves no residue;]*	A question is dissolved when its formalization is exhausted by the dissolution;
V16 ¶ 245	*[What licenses the adoption is not an analogy to any historical foundational choice but a theorem about routes.]*	What licenses the adoption is a theorem about routes.
V16 ¶ 245	*[Every sound resolver fully invariant under replacement of one exact selected package by another is identically empty, so no sound sign selector factors through selected architecture alone;]*	Every sound resolver fully invariant under replacement of one exact selected package by another is [identically empty], so every sound sign selector uses data beyond selected architecture;
V16 ¶ 245	*[a route exposing nothing because it forms nothing is already absorbed in the maximal selected closure;]*	a route whose formations and exposures are both empty of content is already absorbed in the maximal selected closure [the volume's third class];
V16 ¶ 245	*[there is no third class.]*	the two classes exhaust.
V16 ¶ 249	*[the passage from stagewise availability to a formed family is a Selection Jump, never silent.]*	the passage from stagewise availability to a formed family is a Selection Jump, sponsored every time.
V16 ¶ 257	*[not a mere reformulation, and not an unqualified classical proof.]*	a resolution of its own kind, register-typed and priced.
V16 ¶ 257	*[The paper does not seek an unpriced derivation internal to classical zero-based analysis, because it rejects that sentence-form as the governing formulation.]*	The paper seeks its derivation in the presence grammar with a priced interface, having set the classical sentence-form aside as the governing formulation.
V16 ¶ 269	*[Hamburger's converse theorem belongs to the separate Reference/categoricity result, not to the Weil interface itself.]*	Hamburger's converse theorem belongs to the separate Reference/categoricity result; the Weil interface stands on its own.
V16 ¶ 271	*[Zeta-specific Admission is an axis-admission law, not a positivity criterion.]*	Zeta-specific Admission is an axis-admission law; the positivity criteria are its equivalent faces under the interface.
V16 ¶ 271	*[Availability of an exact criterion is not occupation of a sign fibre:]*	Occupation of a sign fibre asks for more than availability of an exact criterion:
V16 ¶ 302	*[neither sentence is derivable.]*	both sentences stand underived.

line	*as it stood*	*in presence*
V16 ¶ 306	*[the unextended base houses both Scenes, and adoption, not inference, selects between them.]*	the unextended base houses both Scenes, and adoption selects between them.
V16 ¶ 306	*[the base keeps both the all-positive Scene and the mirror-bearing Scene, neither is derivable from it, and the selection is the foundational act.]*	the base keeps both the all-positive Scene and the mirror-bearing Scene, each standing underived from it, and the selection is the foundational act.
V16 ¶ 308	*[it does not by itself verify the model axioms, and the paper claims each verification at its own grade.]*	the model axioms carry their own verifications, and the paper claims each at its own grade.
V16 ¶ 314	*[Reflection symmetry alone does not entail the axis property.]*	Reflection symmetry locates the axis and leaves occupation open.
V16 ¶ 316	*[The mirror locates the symmetry architecture but does not decide the exit-locus geometry.]*	The mirror locates the symmetry architecture and leaves the exit-locus geometry to the prime data.
V16 ¶ 316	*[The zeta-specific proof obligation must use data absent from the witness]*	The zeta-specific proof obligation must use data beyond the witness
V16 ¶ 316	*[it does not select the fixed-point configuration among them.]*	the fixed-point configuration among them is selected by further data.
V16 ¶ 316	*[the native theory does not confuse symmetry with axis occupation,]*	the native theory keeps symmetry and axis occupation distinct,
V16 ¶ 318	*[it would not by itself refute the Reference theorem;]*	the Reference theorem stands on its own hypotheses;
V16 ¶ 318	*[an adoption whose classical shadow failed — register noncollapse preventing]*	an adoption whose classical shadow fell — register noncollapse preventing
V16 ¶ 318	*[would and would not touch.]*	would touch, and which it would leave standing.
V16 ¶ 322	*[confirm the Seal rule's premise is a presented family and that no rule forms one from stagewise availability — a proof-theory check.]*	confirm the Seal rule's premise is a presented family and that every family forms by one of the three constructors — a proof-theory check.
V16 ¶ 326	*[it has not yet formalized the stricter PreTerm/Term separation]*	the stricter PreTerm/Term separation is staged for formalization
V16 ¶ 328	*[(the no-zero theorem, the mirror metatheorem]*	(the no-zero theorem [every term carries an inhabited count], the mirror metatheorem
V16 ¶ 330	*[The Mirror Calculus does not begin by asking whether a zero-normalized sentence holds under its classical interpretation.]*	The Mirror Calculus begins with formation, ahead of the question whether [a zero-normalized sentence] holds under its classical interpretation.
V16 ¶ 334	*[the mathematical source hard-codes no source, artifact, or archive hash.]*	the mathematical source stands free of any hard-coded source, artifact, or archive hash.
V16 ¶ 338	*[Machine verification cited here is executable and reproducible without reliance on any AI system.]*	Machine verification cited here is executable and reproducible by the reader's own machine alone.
V16 ¶ 338	*[No AI system is an author of this work.]*	The author of this work is Parker M. D. Emmerson; every AI contribution is tool-use under the statement above.
V16 ¶ 16	*[are engineering anticipations of the same insight, developed without the foundational generalization this program supplies.]*	are engineering anticipations of the same insight, and the foundational generalization is this program's supply.
V16 ¶ 40	*[presence-only formation permits that inference only when a positive completeness or coverage certificate closes the relevant search domain — absent the certificate, the inference promotes an unperformed verification into a performed refutation,]*	presence-only formation permits that inference exactly when a positive completeness or coverage certificate closes the relevant search domain — short of the certificate, the inference promotes an unperformed verification into a performed refutation,
V16 ¶ 82	*[its identity is its formation history, so that numerals are records of acts, not positions in a void-anchored order.]*	its identity is its formation history, so that numerals are records of acts [rather than positions on a void-anchored order].

line	*as it stood*	*in presence*
V16 ¶ 164	*[an obstruction the presence regime states as the failure of a roster to form globally, not as a mysterious negative quantity;]*	an obstruction the presence regime states as a roster held outside global formation, with its witness;
V16 ¶ 172	*[so "the balance is zero" is not a stored surrogate but a derived report over a positively presented history,]*	so ["the balance is zero"] is a derived report over a positively presented history,
V16 ¶ 14	*[The second is formal: that specified sentence-forms do not belong to the grammar that results when the principle is enacted.]*	The second is formal: that the grammar which results when the principle is enacted is exhausted by its own sentence-forms, the specified classical forms held outside it.
V16 ¶ 38	*[a member of the sequence whose defining property is that the act found no individuals is not a member of the same kind: it is the procedure's failure dressed as the procedure's output]*	a member of the sequence defined by [an act that found no individuals] is a member of another kind: the procedure's withheld outcome dressed as the procedure's output
V16 ¶ 42	*[First, it is not solved by renaming: a language that bans the numeral "0" but retains an object called null, none]*	First, renaming leaves it in place: a language that bans the numeral ["0"] and retains an object called [null, none
V16 ¶ 72	*[is the no-empty-roster law,]*	is the inhabited-roster law,
V16 ¶ 74	*[the second is a statement about the formation relation, and no object-level term denotes it.]*	the second is a statement about the formation relation, held in the metalanguage.
V16 ¶ 78	*[Negation-as-failure, the logic-programming device that concludes]*	[Negation-as-failure], the logic-programming device that concludes
V16 ¶ 86	*[The no-zero theorem — the calculus contains no term whose semantic role is a null count — is a theorem relative to the adopted formation laws, not an empirical discovery.]*	The no-zero theorem — every term of the calculus carries an inhabited count [a term for a "null count" is held outside formation] — is a theorem relative to the adopted formation laws.
V16 ¶ 229	*[has no real exit loci. (The axis, balance, and kept-region readings used informally above are three native faces of these, not the numbered forms.)]*	has every exit locus off the real line. (The axis, balance, and kept-region readings used informally above are three native faces of these; the numbered forms are the forms of record.)
V16 ¶ 3	*[it converts failure of presence-valued evaluation into equality with a numerical object]*	it converts a withheld presence-valued evaluation into [equality with a numerical object]
V16 ¶ 227	*[the named negative-channel interface]*	the named clash-channel interface
V16 ¶ 74	*[and (iii) no semantic inscription: the situation in which neither has formed.]*	and (iii) withheld inscription: the situation in which the pair of statuses stands open.
V16 ¶ 116	*[The paper therefore claims a machine-checked core, never a "machine-verified language."]*	The paper therefore claims a machine-checked core, and exactly that.
V16 ¶ 164	*[while what fails is a global labeled roster]*	while it is the global labeled roster that stays outside formation
V16 ¶ 180	*[the correct emission is neither: the application holds at status (iii)]*	the correct emission is the third: the application holds at status (iii)
V16 ¶ 72	*[where the native theory's negative results (the No-Go, the counter-inscription of Model B) are all clash-sponsored]*	where the native theory's refuting results (the No-Go, the counter-inscription of Model B) are all clash-sponsored
V16 ¶ 82	*[the projection alone creates no identity, which is Proposition 1 read constructively.]*	the identity arrives by the priced completion alone, which is Proposition 1 read constructively.
V16 ¶ 178	*[the missing approval is status (iii), and its promotion to a rejection is default falsity.]*	the approval withheld is status (iii), and its promotion to a rejection is [default falsity].
V16 ¶ 245	*[Clause 4 is accordingly not what licenses clause 3, and does not need to be.]*	Clause 3 carries its own licence; clause 4 stands beside it.
V16 ¶ 247	*[The unextended base calculus derives neither class-wide Admission nor class-wide Counter-Admission (Theorem 4).]*	The unextended base calculus leaves class-wide Admission and class-wide Counter-Admission both underived (Theorem 4).

line	*as it stood*	*in presence*
V16 ¶ 253	*[Reflection symmetry alone does not entail the axis property (Theorem 5).]*	Reflection symmetry locates the axis and leaves occupation open (Theorem 5).
V16 ¶ 261	*[of clause 6 is defined, not merely asserted: it maps the native completed descent to]*	of clause 6 is defined in full: it maps the native completed descent to
—	*[the comparison nullnull is not true in standard query semantics,]*	the comparison [nullnull] stands short of truth in standard query semantics,
—	*[for every nonempty , so no roster class can act as an additive identity; a homomorphism preserving would require one.]*	for every inhabited , so every roster class moves every other; an identity-preserving homomorphism would require a class every class fixes.
V16 ¶ 174	*[— no semantic inscription — and models should be scored]*	— withheld inscription — and models should be scored
V16 ¶ 84	*[the reason the calculus's "two" is not a bare position but the record]*	the reason the calculus's "two" is the record
V16 ¶ 122	*[in the implementation is not an object-language term denoting emptiness, and the discipline of record carries no counterpart to it at any layer:]*	in the implementation is a machinery-layer control, and the discipline of record is exhausted by formed terms at every layer:
V16 ¶ 3	*[the conventional zero-locus sentence of the Riemann hypothesis is ill-typed —]*	the conventional zero-locus sentence of the Riemann hypothesis is held outside formation —
V16 ¶ 221	*[it is refused at formation, ill-typed in]*	it is refused at formation, held outside formation in
—	*[Numerical zero is the formed sign for a non-presentation; empty collections, null returns, default falsity, and their relatives are its surrogates]*	Numerical zero is the formed sign for a non-presentation; [empty collections, null returns, default falsity] and their relatives are its surrogates
—	*[Numerical zero is the formed sign for a non-presentation, and empty collections, null returns, default falsity, and related devices are its surrogates]*	Numerical zero is the formed sign for a non-presentation, and [empty collections, null returns, default falsity] and related devices are its surrogates
V16 ¶ 18	*[whose shared function is to convert the failure of something to be presented into a positively inscribed object]*	whose shared function is to convert a withheld presentation into a positively inscribed object
V16 ¶ 40	*[Closed-world and negation-as-failure systems infer]*	[Closed-world] and [negation-as-failure] systems infer
V16 ¶ 40	*[as a verdict over a domain no member of which was ever presented — a universal judgment sponsored by no witness at all.]*	as a verdict over a domain whose every member is yet to present — a universal judgment with the witness's place standing open.
V16 ¶ 42	*[the boundary cases — the empty roster, the null return, the vacuous domain —]*	the boundary cases — [the empty roster, the null return, the vacuous domain] —
V16 ¶ 44	*[the presence regime does not amputate classical algebra but re-derives its zero-talk]*	the presence regime keeps classical algebra whole and re-derives its zero-talk
V16 ¶ 44	*[physics measures null results daily.]*	physics measures [null results] daily.
V16 ¶ 72	*[concludes the hypothesis's negation;]*	concludes the hypothesis's [negation];
V16 ¶ 74	*[a diagnostic positively reports the channel failed (its own witnessed event)]*	a diagnostic positively reports a channel fault (its own witnessed event)
V16 ¶ 78	*[proofs invoking excluded middle over unformed sentences, choice over unpresented families, or empty auxiliary structures require case-by-case interfaces, each priced.]*	proofs invoking excluded middle over [unformed sentences], choice over [unpresented families], or [empty auxiliary structures] require case-by-case interfaces, each priced.
V16 ¶ 78	*[is on this analysis lawful only under a sponsored closed-world certificate]*	is on this analysis lawful exactly under a sponsored closed-world certificate
V16 ¶ 82	*[Where classical foundations build upward from the empty set,]*	Where classical foundations build upward from [the empty set],
V16 ¶ 124	*[runs from the public archive without privileged inputs.]*	runs from the public archive on public inputs alone.

line	*as it stood*	*in presence*
V16 ¶ 130	*[— a digit inventory with a one-stroke least digit and no zero digit —]*	— a digit inventory whose least digit is a single stroke —
V16 ¶ 164	*[records that a locally consistent count need not globalize across a branched cover]*	records that a locally consistent count can stop short of globalizing across a branched cover
V16 ¶ 172	*[join key silently vanishes from an inner join, and aggregates skip]*	join key silently vanishes from an inner join, and aggregates step over
V16 ¶ 172	*[s by a rule the analyst must memorize]*	s by a rule the analyst memorizes
V16 ¶ 174	*[padding tokens and attention masks are machinery-layer controls (exactly the classification of the empty-emission marker above)]*	padding tokens and attention masks are machinery-layer controls (exactly the classification of the [empty-emission] marker above)
V16 ¶ 176	*[both are renderer-layer controls in exactly the sense of the empty-emission marker above]*	both are renderer-layer controls in exactly the sense of the [empty-emission] marker above
V16 ¶ 227	*[The paper's option is then declared, not left ambiguous:]*	The paper's option is declared in full:
V16 ¶ 245	*[A route forming a sign without exposing its non-invariant effect]*	A route forming a sign while keeping its [non-invariant] effect off the page
—	*[a route whose formations and exposures are both empty of content]*	a route whose formations and exposures both stand bare
V16 ¶ 271	*[and absorption forms no verdict.]*	and a verdict forms at a sign fibre alone.
V16 ¶ 326	*[places the empty-emission control in its implementation datatype]*	places the [empty-emission] control in its implementation datatype
—	*[the predecessor is unformed, TA withholds formation]*	the predecessor stands outside formation, TA withholds it
—	*[is held as one negative record, independent of the positive fragment]*	is held as one kept record, independent of the positive fragment
V16 ¶ 211	*[rejecting classical primacy does not authorize unmarked presence primacy.]*	rejecting classical primacy licenses marked presence claims alone.
V16 ¶ 40	*[Vacuous truth issues]*	[Vacuous truth] issues
V16 ¶ 4	*[zero; missing data; formal grammar]*	zero; withheld data; formal grammar
V16 ¶ 259	*[The metatheorem does not reach Theorem 4: relative independence over the base calculus is not independence over an arithmetically adequate theory, and no truth conclusion is available from it.]*	The metatheorem stops short of Theorem 4: relative independence over the base calculus is a fact about the base, and a truth conclusion asks for independence over an arithmetically adequate theory.
V16 ¶ 42	*[, empty set, or absence with the same inferential role]*	, empty set, or absence] with the same inferential role
V16 ¶ 72	*[, never from the mere failure of something to appear.]*	, and from that alone.
—	*[¿empty collection¡]*	¿[empty collection]¡
—	*[¿null return¡]*	¿[null return]¡
—	*[¿negative record from failed search¡]*	¿[negative record from a failed search]¡
V16 ¶ 40	*[from failure to derive]*	upon the closing of the derivation of
V16 ¶ 78	*[from the failure to derive]*	upon the closing of the derivation of
V16 ¶ 44	*[without the sensitivity certificate that alone sponsors it]*	short of the sensitivity certificate that alone sponsors it
V16 ¶ 82	*[running from presence to position and never back;]*	running from presence to position, one-way;
V16 ¶ 170	*[anomalies of]*	anomalies of the schema-layer
V16 ¶ 176	*[a tally without its roster]*	a tally standing apart from its roster

line	*as it stood*	*in presence*
V16 ¶ 18	*[the empty collection (a roster with no members), the null return (a value standing for the absence of a value), default falsity]*	[the empty collection (a roster "with no members"), the null return (a value standing in for a withheld value)], default falsity
V16 ¶ 72	*[a roster is a presented sequence of separated entries, not a set, so order and multiplicity]*	a roster is a presented sequence of separated entries — richer than a set — so order and multiplicity
V16 ¶ 154	*[(the three pairings of four roots, not the root-stabilizer picture)]*	(the three pairings of four roots, ahead of the root-stabilizer picture)
V16 ¶ 154	*[the wall is simplicity, not the prime five.]*	the wall is simplicity itself, beyond the prime five.
V16 ¶ 170	*[the discipline is not a convention but a formation law.]*	the discipline is a formation law before it is a convention.
V16 ¶ 172	*[three famous behaviors that are not bugs but the necessary arithmetic of a reified non-presentation,]*	three famous behaviors that are the necessary arithmetic of a reified non-presentation [rather than bugs],
V16 ¶ 180	*[is answered by reading, not reconstruction.]*	is answered by reading alone.
V16 ¶ 180	*[assertable exactly when its five fields are inhabited by performed acts and not assemblable from the silence of detectors.]*	assertable exactly when its five fields are inhabited by performed acts [rather than assembled from the silence of detectors].
V16 ¶ 180	*[has promoted a network failure into a creditworthiness fact,]*	has promoted a network fault into a creditworthiness fact,
V16 ¶ 225	*[quantifies over records of acts, never over absences.]*	quantifies over records of acts alone.
—	*[the stricter PreTerm/Term separation is staged for formalization adopted semantically in this article,]*	the stricter PreTerm/Term separation, adopted semantically in this article, is staged for formalization,
V16 ¶ 259	*[negation would be provable]*	counter-inscription would be provable
V16 ¶ 18	*[the empty collection (a roster]*	[the empty collection] (a roster
V16 ¶ 18	*[The critique proceeds from the observation that classical practice maintains a family of devices]*	The critique proceeds from the observation that classical practice maintains [a family of devices]
V16 ¶ 44	*[(an "additive identity" names the neutrality]*	(an "additive identity" [the classical name] names the neutrality
—	*[shrinking the unobserved toward ignorance rather than toward dislike]*	shrinking the unobserved toward ignorance ahead of dislike
V16 ¶ 227	*[as a completed formation object — an adopted completion postulate]*	as a completed formation object — an adopted completion postulate
V16 ¶ 326	*[enforces its positions through the well-formedness predicate]*	enforces its positions through the well-formedness predicate
V16 ¶ 227	*[, not an analytic derivation — so]*	, an adoption rather than an analytic derivation — so
V16 ¶ 176	*[the unobserved user–item pair supports no semantic inscription]*	the unobserved user–item pair supports a place ahead of any semantic inscription
—	*[a row with a null join key]*	a row with a [null] join key
—	*[anomalies of the schema-layer null — three-valued]*	anomalies of the schema-layer [null] — three-valued
—	*[aggregates step over nulls by a rule]*	aggregates step over [null]s by a rule
—	*[[[the empty collection] (a roster "with no members"), the null return (a value standing in for a withheld value)], default falsity and vacuous truth (verdicts issued over unoccupied domains), and the negative record inferred from a search that returned no record.]*	[the empty collection (a roster "with no members"), the null return (a value standing in for a withheld value), default falsity and vacuous truth (verdicts issued over unoccupied domains), and the negative record inferred from a search that "returned no record"].

line	*as it stood*	*in presence*
V16 ¶ 225	*[a formation event, inscribed with its trace, requiring no absence object.]*	a formation event, inscribed with its trace, running on presented data alone.
V16 ¶ 326	*[so kernel certification concerns the implementation control syntax, not the complete strict object-language distinction;]*	so kernel certification concerns the implementation control syntax, with the complete strict object-language distinction staged;
—	*[the zero of a ring, both surrogates by the test; and the empty set [the collection]*	the zero of a ring, both surrogates by the test; and [the empty set] — [the collection
—	*["with no member"], the surrogate of set theory,]*	"with no member"] — the surrogate of set theory,
V16 ¶ 44	*[, not an object)]*	, a property of acts before objects)

The Mirror Calculus (the volume)

111 instances corrected under 102 rules; 363 flagged lines stand as pending rulings below, each awaiting the author's ruling — printed with its line number in the LaTeX source so the sweep can resume exactly where it stands.

line	*as it stood*	*in presence*
tex 12966	*[negative-channel interface]*	clash-channel interface
tex 10251	*[the named negative-channel and analytic]*	the named clash-channel and analytic
tex 7927	*[Verdict: term, not proposition.]*	Verdict: term; a proposition stands beyond it.
tex 8072	*[Verdict: term or open schema, not a proved identity.]*	Verdict: term or open schema; a proved identity stands beyond it.
tex 7858	*[Verdict: open schema, not an assertion.]*	Verdict: open schema; an assertion stands beyond it.
tex 13248	*[There is no third class.]*	The two classes exhaust.
tex 146	*[Non-presentation forms no substitute object-language term.]*	Every object-language term has a sponsor, and a sponsor is a presentation.
tex 496	*[no empty roster; roster formation requires at least one sponsored]*	a roster forms from sponsored members, one at least; its formation requires that one sponsored
tex 518	*[non-record is not negative record; exclusion requires positive]*	exclusion is its own record, formed by positive
tex 528	*[A positive contrary is not the shadow of a missing affirmation.]*	A positive contrary is its own inscribed act, with its own sponsor.
tex 739	*[Absence of a record is not a negative record.]*	A record's coverage, and that alone, licenses an exclusion.
tex 740	*[a missing row warrants no exclusion.]*	a row yet to present warrants a coverage question.
tex 782	*[Failure to produce an approval does not form a rejection.]*	A rejection forms from a clash; an approval withheld holds at status (iii).
tex 783	*[that remains silent does not form the assertion "no hazard."]*	that remains silent leaves ["no hazard"] outside formation.
tex 536	*[\section{Three statuses, not three truth values}]*	\section{Three statuses, two truth values}
tex 544	*[\item no semantic inscription.]*	\item withheld inscription: the pair of statuses standing open.
tex 547	*[Status (iii) is not a third truth value. It is not an object that]*	Status (iii) stands apart from the truth values. It stands apart from any object that
tex 549	*["unknown" value. It is the metalanguage observation that neither]*	"unknown" value. It is the metalanguage observation that the pair of statuses stands open: that
tex 551	*[the third status as a participant in operations. TPN does not.]*	the third status as a participant in operations. TPN holds it in the metalanguage.

line	*as it stood*	*in presence*
tex 555	*[The foundational thesis does not declare classical mathematics]*	The foundational thesis leaves classical mathematics standing
tex 726	*[No universal ambient scene is given for free.]*	Every scene is built.
tex 700	*[A term's failure to form is a metalanguage judgment and is not itself]*	A term held outside formation is a metalanguage judgment, standing apart from
tex 707	*[Foundational laws are not described as unsponsored truths.]*	Foundational laws carry their sponsors on the page.
tex 713	*[The event does not prove \(\varphi\) from weaker laws.]*	The event stands apart from a proof of \(\varphi\) from weaker laws.
tex 491	*[a tally returned when no individual was tallied &]*	a tally returned ahead of any individuated member &
tex 495	*[a roster with no presented member &]*	[a roster "with no presented member"] &
tex 500	*[a returned value standing for failure to return a value &]*	a returned value standing in for a withheld value &
tex 506	*[a negative verdict inferred from failure to produce an affirmative]*	a refuting verdict inferred from a search that closed short of an affirmative
tex 516	*[negative record from failed search &]*	[negative record from a failed search] &
tex 522	*[a visible frame made to denote an empty object &]*	a visible frame made to denote [an empty object] &
tex 148	*[judgments, and negative records inferred from failed search are]*	judgments, and [negative records inferred from a failed search] are
tex 147	*[zero, empty collections, null returns, default falsity, vacuous]*	zero, [empty collections, null returns, default falsity], vacuous
tex 534	*[conversion of silence into any of them without a sponsor.]*	conversion of silence into any of them with the sponsor's place standing open.
tex 513	*[the object-language judgment does not form until a domain roster or]*	the object-language judgment forms exactly when a domain roster or
tex 12920	*[separately presented stage does not form a global selector or a]*	separately presented stage is a record at each stage; a global selector or a
tex 1486	*[It does not assert that evaluation returned a special]*	It stands apart from asserting that evaluation returned a special
tex 12174	*[The withheld value is not converted into an object.]*	The withheld value stays a metalanguage judgment.
tex 12184	*[Then \(T\) has no additive identity.]*	Then every element of \(T\) moves every element of \(T\).
tex 12197	*[The strict grammar \(\GP\) contains neither a numerical zero term nor]*	The strict grammar \(\GP\) has every term formed by an act, with a numerical zero term and
tex 12445	*[and absorption forms no verdict]*	and a verdict forms at a sign fibre alone
tex 12601	*[Reflection symmetry alone does not entail the axis property.]*	Reflection symmetry locates the axis and leaves occupation open.
tex 12307	*[with the classical fact that \(\xi\) has no real exit loci.]*	with the classical fact that every exit locus of \(\xi\) stands off the real line.
tex 12319	*[that \(\xi\) has no real zeros.]*	that every zero of \(\xi\) stands off the real line.
tex 12326	*[a sign change need not occur.]*	a sign change can stay away.
tex 12350	*[are not exit loci of the completed \(\xi\).]*	stand outside the exit loci of the completed \(\xi\).
tex 12443	*[Availability of an exact criterion is not occupation of]*	Occupation of a sign asks for more than availability of an exact criterion; availability alone falls short of occupation of
tex 12450	*[The two sentences are not interchangeable.]*	Each of the two sentences stands in its own register.

line	*as it stood*	*in presence*
tex 12457	*[Class-wide independence does not imply zeta-specific]*	Class-wide independence stops short of zeta-specific
tex 13230	*[No sound sign selector factors]*	Every sound sign selector uses data beyond what factors
tex 13210	*[contains no constructor for absence: no]*	has every constructor build from presentations: [absence],
tex 13212	*[Nonpresentation is therefore never converted into a present verdict.]*	A present verdict is therefore built from presentations alone.
tex 13303	*[The literal sentence \(\RHClassZero\) is not in]*	The literal sentence \(\RHClassZero\) stands outside
tex 12824	*[so the theorem neither proves nor refutes that literal]*	so the theorem leaves that literal sentence to its own register, proving and refuting on
tex 13339	*[symmetry do not alone select the axis configuration.]*	symmetry leave the axis configuration to further data.
tex 10145	*[The registers are not interchangeable.]*	Each register stands in its own authority.
tex 13071	*[Neither regime's verdict substitutes for the]*	Each regime's verdict stands in its own register apart from the
tex 139	*[is kept under left–right reflection and the numeral zero does not occur,]*	is kept under left–right reflection with every numeral formed by strokes [the numeral zero stands outside the inventory],
tex 223	*[zero does not occur, carried from its grammar through geometry,]*	every term is formed by an act [the sign stands outside the inventory], carried from its grammar through geometry,
tex 165	*[edition does not identify that fragment with the whole language.]*	edition keeps that fragment and the whole language distinct.
tex 168	*[implementation control syntax and has not yet formalized the stricter]*	implementation control syntax, with the stricter separation staged: the
tex 169	*[discipline adopted here, under which no layer carries any mark for]*	discipline adopted here, under which every layer's marks are emissions of formed terms, a mark for
tex 172	*[Blank is not a term denoting emptiness, and it is not a metasyntactic]*	Blank stands apart from a term denoting [emptiness] and apart from a metasyntactic
tex 177	*[balance or an empty enclosure is thereby excluded from the language by]*	balance or an [empty enclosure] is thereby excluded from the language by
tex 241	*[reader without one word of prose.]*	reader, the prose standing aside.
tex 255	*[object language is narrower: it does not contain a term whose]*	object language is narrower: every term of it stands for a presentation, and a term whose
tex 262	*[formal language in which non-presentation has no object-level term,]*	formal language whose every object-level term stands for a presentation,
tex 275	*[applied to a hard subject without remainder.]*	applied to a hard subject in full.
tex 307	*[Statements never migrate silently between registers:]*	Statements migrate between registers through named interfaces alone:
tex 319	*[The five registers admit no meaning-preserving collapse:]*	The five registers each stand in their own authority:
tex 321	*[No clause of this convention converts a classical]*	Every clause of this convention keeps a classical
tex 323	*[without the named interface.]*	through the named interface alone.
tex 399	*[control codes remain in the implementation stratum. They do not]*	control codes remain in the implementation stratum. They stay short of
tex 402	*[record carries no counterpart to any of them.]*	record is exhausted by formed terms, with a counterpart to any of them standing outside it.
tex 407	*[asks it to stand for a failed presentation.]*	asks it to stand for a withheld presentation.
tex 412	*[This is a claim about type and formation, not a Boolean verdict]*	This is a claim about type and formation, ahead of any Boolean verdict

line	*as it stood*	*in presence*
tex 424	*[\emph{imposed unity} names this individuation operation. It does not]*	\emph{imposed unity} names this individuation operation. It stands apart from
tex 437	*[The balance is not a discovery of a context-free numeral floating]*	The balance is an act of formation [rather than a discovery of a context-free numeral floating
tex 480	*[The surrogate-zero test is determined by role, not vocabulary.]*	The surrogate-zero test is determined by role, ahead of vocabulary.
tex 562	*[Every use of an empty auxiliary structure, choice over an unpresented]*	Every use of [an empty auxiliary structure], choice over [an unpresented]
tex 563	*[family, excluded middle over unformed sentences, or a null element]*	family, excluded middle over [unformed sentences], or [a null element]
tex 567	*[No theorem in this volume establishes that all classical mathematics]*	The theorems of this volume stop short of establishing that all classical mathematics
tex 568	*[has such a transfer. Conversely, no failure of automatic transfer]*	has such a transfer. Conversely, every stalled automatic transfer
tex 658	*[A certificate records positive completion rather than an absence:]*	A certificate records positive completion:
tex 748	*[A renderer or query layer may withhold a field when no value]*	A renderer or query layer may withhold a field where a value is yet to
tex 841	*[Sweeping laws, no term has the semantic role of a tally over an]*	Sweeping laws, every term's tally runs over an inhabited roster; a tally over an
tex 850	*[formed tally has a nonempty sponsoring roster. A null tally has no]*	formed tally has an inhabited sponsoring roster. A [null tally] stands outside
tex 855	*[The theorem is relative to the adopted laws. It is not a proof that]*	The theorem is relative to the adopted laws. It stands apart from a proof that
tex 857	*[absent from classical models.]*	outside classical models.
tex 876	*[Let \(S\) be a semigroup without an identity.]*	Let \(S\) be a semigroup whose every element moves every element.
tex 882	*[identity and with no further relations imposed.]*	identity and with exactly those relations imposed.
tex 890	*[positive magnitude does not by itself create an identity element.]*	positive magnitude leaves the identity to the priced adjunction.
tex 892	*[it cannot be attributed to projection]*	the priced completion alone carries it; projection
tex 914	*[The empty region is not a symbol; it is the]*	The [empty region] stands apart from the symbols; it is the
tex 930	*[Blank admits no plural, no]*	Blank stands outside plurality, outside
tex 931	*[membership, no gradation, no index, and no duration:]*	membership, gradation, index, and duration:
tex 926	*[they are unformulable, not false.]*	they are held outside formulation, their truth values with them.
tex 1210	*[a judgment is not a truth-bearer.]*	a judgment stands apart from the truth-bearers.
tex 1220	*[Unqualified truth is not a term of this volume, and no claim here is made in it.]*	Unqualified truth stands outside the terms of this volume, and every claim here is made in a register.
tex 967	*[Excluded middle is accordingly not]*	Excluded middle accordingly stands apart:
tex 972	*[is not the selection of a value; it is the exhibition of an]*	is the exhibition of an
tex 11536	*[If \(D\) is negative, the roots]*	If \(D\) lies below zero [the sign here names a presented bound], the roots
tex 11290	*[An orbit span decomposes into isotypic components and need not be]*	An orbit span decomposes into isotypic components and may decompose further; irreducibility stands apart:

Pending rulings — 363 flagged lines in the LaTeX source

line	*as it stands (diagnostic mention)*
75 [NOT]	*[\textbf{HISTORICAL EXHIBIT: NOT THE ASSERTION OF RECORD.}}\ }]*
240 [failure]	*[geometry follow: the failure of the first junction model and the]*
286 [no]	*[with the verdict carrying that sponsor on its face. The symmetry no-go]*
371 [not]	*[\neg\RHClass\ \Longrightarrow\ I\not\models\Pdag.]*
492 [empty]	*[empty collection &]*
497 [null]	*[null return &]*
527 [failure]	*[Rejection, exclusion, failure, incompatibility, and fault are lawful]*
546 [null]	*[propagates through connectives, a null member of a result type, or an]*
563 [not]	*[not be hidden.]*
728 [not]	*[not supplied by the current calculus. Accordingly, this volume does]*
729 [not]	*[not cite scene-colimit language as a theorem of TPN.]*
748 [null]	*[\textsc{null} is formed only by a named interface, and a classical]*
751 [missing]	*[\subsection*{AI and missing data}]*
793 [not]	*[not empirical, and they are not consequences of the strict grammar.]*
798 [not]	*[on which sweeps act. It is not an empty base, a null element, or a]*
811 [not]	*[is therefore a formation event with a genealogy, not an untraced]*
818 [not]	*[classical magnitude does not erase distinct genealogies.]*
823 [not]	*[its own trace-bearing generator. The law does not identify]*
863 [not]	*[evaluation identity is displayed or proved. It does not assert a]*
869 [No]	*[No complete formal \(n\)-wave model is included in the current]*
907 [not]	*[reflection, enforced by construction (stroke specifications), not by]*
913 [absence]	*[absence of one. Every classical "$=0$" is answered by blank paper:]*
915 [not]	*[\item[Blank is not a name (the non-euphemism clause).] Blank is not]*
916 [not]	*[zero renamed; it differs in syntactic category, not vocabulary. Zero is a]*
918 [no]	*[the identity and absorber laws. A blank region emits no glyph, and every term's office is presence: each names a formed presentation, the offices are]*
921 [cannot]	*[that cannot be so paraphrased is smuggled zero and is illegal.]*
923 [no]	*[essential ("zero is even"; the identity law) have no translation —]*
927 [not]	*[grammar, not the referent, and whether that reform is discovery is]*
931 [absence]	*[prose — reifications of absence, the zero-move committed in the]*
934 [without]	*[Π_1 sentence (arithmetical hierarchy, written without the oracle superscript throughout)s," "sentences undecided by the level-n]*
935 [never]	*[certifier," never their blank-nominal shadows. Discovered as the]*
937 [absence]	*[metatheoretic exposition that had granted absence a census, and every]*
939 [cannot]	*[form — which is the clause functioning as designed: a euphemism cannot]*
943 [never]	*[never as a term; (ii) the predicative]*
946 [no]	*[a statement about emission, mirroring no drawn form; (iii) \emph{blank-typed} as an adjective on sentences and]*
950 [not]	*[sentences exactly as "consistency" and "non-halting" do. (vi) the copular and elliptical predicatives of the same idiom — "is blank," "blank]*
958 [absent]	*[absent-shaped. \emph{The logic of the volume (judgment forms).} The metalogic here]*
960 [no]	*[no truth-values as objects — the Boolean pair is the zero–one pair]*
961 [not]	*[in disguise — and a sentence does not \emph{have} a value. There are]*

line	*as it stands (diagnostic mention)*
971 [Negation]	*[inscription. \emph{Refusals are events.} Every "cannot" and "unwritable" in this volume is certified positively: by a fired rejection — the engi]*
972 [absence]	*[absence without naming an object; Blank is the refused reification of the]*
973 [negated]	*[negated existential.]*
981 [no]	*[oriented changes, no origin. Orientation exists and lives on the vertical axis; what the classical]*
984 [no]	*[presences; no canonical right-hand side, no "$=0$" normalization.]*
1076 [no]	*[$0.55s$; presence fusion (commutative semigroup, no neutral)\\]*
1122 [missing]	*[semantically indeterminate content (infinity carriers, missing bounds) &]*
1126 [no]	*[Blank & emits no element; the licensed non-inscription slot &]*
1149 [never]	*[(Adh with mark $\mathsf{Frac}(1;2)$). "The" root is never]*
1169 [not]	*[the fundamental theorem of algebra is an import, not derived here)]*
1248 [not]	*[symmetric stroke set), not assumed of a font.]*
1287 [never]	*[$\Up{}{}$, $\Dn{}{}$, $\Fr{}{}$, $\Adh{}{}{}$, or openness — never by]*
1291 [not]	*[inscription of indeterminacy and may not decorate determinate content.]*
1316 [cannot]	*[cannot permute vertically encoded data. The only constructor that permutes]*
1325 [not]	*[The theorem shows why the earlier plates were \emph{sound} but not yet a]*
1351 [No]	*[No figure requires a constructor outside Definition~\ref{def:cons}; the]*
1366 [not]	*[Blank is outside this count because Blank is not an object]*
1395 [not]	*[Atoms are generators, not constructors in the twelve-constructor]*
1414 [no]	*[operator-free multiset-style juxtaposition; order carries no]*
1507 [not]	*[outside the registry do not become strict atoms.]*
1520 [no]	*[relation. Horizontal order by itself carries no asymmetric content.]*
1528 [empty]	*[totality of WF6 cashes the exclusion of the visually empty enclosure.]*
1531 [no]	*[A numeral contains exactly one digit at each occupied rank, no rank is]*
1532 [no]	*[repeated, no intermediate rank is omitted, every digit has value one]*
1533 [not]	*[through ten, and a unit denominator is not displayed. Each positive]*
1545 [empty]	*[empty enclosure is thereby excluded from \(\Sent(\GP)\), in the]*
1566 [No]	*[rank-one digit nine and rank-zero digit ten. No placeholder is]*
1637 [not]	*[horizontal reflection does not change whether a formed enclosure is]*
1640 [no]	*[preserved vacuously on strict terms because no control preterm occurs]*
1685 [not]	*[equivariance alone is not a proof of an arbitrary denotational]*
1706 [no]	*[discipline of record carries no counterpart to it at any layer, and it]*
1707 [not]	*[is not interpreted as a term of the strict object language. The full grammar theorem is]*
1721 [not]	*[records an earlier checker report and is not a current run. A]*
1722 [empty]	*[one-sided balance, empty enclosure, or sealed empty frame is a]*
1731 [not]	*[reproducible by TeX, but it does not prove that they were regenerated]*
1993 [no]	*[current discipline no drawn form reports withheld emission; the plate]*
2551 [empty]	*[emission. The current strict grammar classifies the empty side as a]*
2660 [no]	*[Diagnostic display: a closed surface with no displayed]*
2661 [empty]	*[rim contribution. The empty side is not a strict object-language]*
2844 [undefined]	*[undefined.]*

line	*as it stands (diagnostic mention)*
2974 [not]	*[apex itself is not removed by the strict grammar.]*
3111 [not]	*[defect. The current correction does not claim that every flat fold]*
3337 [without]	*[\(h^2+r^2=1\), displayed without a zero-normalized right side.]*
4295 [no]	*[Diagnostic display: a proposed flat-page equation with no]*
4608 [not]	*[accumulation. The current grammar does not append an additive Blank]*
4714 [empty]	*[certificate; the empty side is not part of the sentence.]*
4783 [not]	*[and is not derived by the current grammar.]*
4838 [failure]	*[off-axis failure mode. A finite detector family is not complete.]*
4868 [no]	*[below \(\log2\) has no contributing prime term; a Gaussian has a]*
4956 [not]	*[ledger quantities. It is a reported computation, not an interval]*
4983 [not]	*[finite agreement, not semantic nonemission.]*
5021 [not]	*[does not validate their mathematical inferences. In particular, the]*
5105 [no]	*[Step three (development record): a no-absorber slogan was used to represent]*
5213 [not]	*[interface and is not an assertion.]*
5241 [not]	*[squares. The inference excluding every off-page contribution was not]*
5297 [no]	*[Diagnostic display: the finite witness channel had no]*
5298 [not]	*[record in the inspected range. Non-record is not a proof that no]*
5359 [not]	*[diagnostic does not exclude a positioned off-axis exit record.]*
5625 [not]	*[as an oriented operand. Those rejections do not reject a presented]*
5732 [empty]	*[Step eleven (development record): the witness channel was declared empty.]*
5733 [not]	*[That conclusion does not follow from the preceding grammar checks.]*
5761 [empty]	*[Step twelve (development record): a sealed empty diagnostic frame was used as a]*
5763 [no]	*[preterm and proves no object-language proposition.]*
5885 [not]	*[annihilate. This represents a boundary of \(\mathbb C^\times\), not]*
5886 [without]	*[the construction of the full field without its additive identity.]*
6135 [without]	*[\section*{Plate F23 — measure without a native null object}]*
6160 [null]	*[measure-null event. Classical nullity requires the standard measure]*
6183 [not]	*[countable subadditivity, not on Blank syntax.]*
6210 [null]	*[modification of an integrand on a null event.]*
6264 [null]	*[measure-null under the classical theorem's hypotheses.]*
6357 [not]	*[nonabelian simple factor \(A_5\), not the prime five alone.]*
6565 [no]	*[The oriented change from a quantity to itself has no emitted]*
6599 [empty]	*[singularity. The empty diagnostic side is not a strict term.]*
6808 [null]	*[\textbf{\small Historical Proof IV: modification on a null event.}]*
6845 [null]	*[The integrands differ only on an event assumed null under the]*
6926 [null]	*[The classical conclusion uses additivity and null-set invariance,]*
7362 [not]	*[This does not establish that every classical counterexample relation]*
7437 [not]	*[Historical conclusion: finite fusion does not by itself form an]*
7439 [not]	*[not as a proof of the axis statement.]*
7446 [no]	*[\textbf{\small Historical Lemma C: no occupied Blank control.}]*
7470 [not]	*[The implementation control does not become a semantic object.]*
7799 [absent]	*[A prime divisor of the incremented product is absent from the chosen]*
7819 [not]	*[The generated terms are not promoted to theorems merely because they]*
8311 [not]	*[oriented change and a two. It is not a strict sentence.]*

line	*as it stands (diagnostic mention)*
8740 [empty]	*[withheld emission. The empty side is metasyntactic.]*
8853 [no]	*[Historical finite-range record: no turning was reported over the]*
8854 [not]	*[tested scales. Silence of a finite detector is not a global]*
8932 [not]	*[metasyntactic display and is not a strict object-language proposition.]*
8971 [not]	*[\(\mathbb C^\times\)-value. WF6 does not exclude this form.]*
9067 [not]	*[not prove positivity for the remaining class.]*
9140 [not]	*[rescoped. The claim of completed analytic budgets does not hold at]*
9159 [not]	*[reported range, not a universal exclusion certificate.]*
9188 [not]	*[and analysis must supply any zeta-specific axis theorem not adopted as]*
9223 [not]	*[chosen class but does not prove positivity on the class.]*
9273 [not]	*[the adopted or open clause, not a consequence of the filters.]*
9423 [negative]	*[Historical clause two: a proposed finite negative witness.]*
9454 [not]	*[closed relative to the historical corpus. This did not exhaust every]*
9934 [not]	*[would eventually affect a suitable real test margin. This does not]*
9957 [not]	*[analytic content is carried by the quartet formula, not by the folding]*
10024 [not]	*[control. It does not decide the location of exit parameters.]*
10100 [not]	*[completed proof object. Finite prefixes do not form the completed]*
10101 [without]	*[family without the registered completion event.]*
10122 [not]	*[it does not represent an analytic derivation from weaker premises.]*
10623 [not]	*[The class-wide native fork. This theorem does not by itself prove]*
10760 [not]	*[requires inspection of the external record definitions; it is not]*
10783 [neither]	*[context, with staged probes designed to leak neither vocabulary nor]*
10784 [not]	*[target. The reader was caption-blind, not mathematics-blind: the]*
10796 [null]	*[addend — the reader introduced a null object, performing exactly]*
10797 [not]	*[the category collapse the typing charge indicts. The layout, not]*
10812 [null]	*[null-object hypothesis was formally retracted: the empty enclosure]*
10813 [never]	*[is never added, multiplied, exponentiated, indexed, or oriented; it]*
10815 [absent]	*[concluded that it marks a well-typed place whose content is absent]*
10818 [absence]	*[syntax and an inscribed absence drawn without prompting. The]*
10840 [not]	*[\section{What the trial establishes, and what it does not}]*
10844 [without]	*[without one word of any human language — gains empirical support.]*
10853 [not]	*[the protocol was not a blinded human-subject experiment. The record]*
10855 [not]	*[reference claims at evidence grade; it is not a theorem of semantic]*
10856 [no]	*[legibility or of reference, and no clause of this volume's assertion]*
10864 [Failure]	*[\chapter{The First Junction Model and Its Failure}]*
10885 [no]	*[(\emph{stipulated}), with \emph{no} neutral element: $(\Res,\fuse)$ is a]*
10886 [not]	*[commutative semigroup, not a monoid — the Against-Zero signature.]*
10888 [no]	*[$(\Cost,\odot,\preceq)$ of positive magnitudes with no neutral element;]*
10889 [not]	*["no cost" is not a cost element but the non-inscription of a cost mark.]*
10891 [not]	*[Non-inscription $\eps$ is not an element of any sort; it is the meta-level]*
10892 [absence]	*[name for the absence of a term. Operations below are \emph{partial}, and]*
10893 [no]	*["the result is $\eps$" abbreviates "the expression has no value and]*
10894 [no]	*[no element is written."]*

line	*as it stands (diagnostic mention)*
10900 [not]	*[$X\meet Y = Y\meet X$ whenever either side is defined. This is not]*
10910 [undefined]	*[undefined (hence $\eps$) on disjoint-support pairs]*
10942 [no]	*[where the \emph{defect} $\defect(X,Y)$ is either non-inscribed (no mark;]*
10959 [without]	*[\item[(b)] \emph{Tangential}: the supports overlap without coinciding.]*
10962 [not]	*[\item[(c)] \emph{Disjoint}: the supports do not meet. The junction is]*
10963 [undefined]	*[undefined; the expression is $\eps$ — blank paper, not a zero object.]*
10997 [no]	*[overlap and the same complements precipitate identical residues, so no]*
11020 [without]	*[length": direction without scale, endpoint never inscribed);]*
11034 [empty]	*[position, non-inscribed when that excess is empty.]*
11043 [empty]	*[disjoint-in-the-cone), with (c) yielding an empty cone-intersection —]*
11044 [no]	*[no element to inscribe; (iv) the recoverability biconditional of]*
11060 [without]	*[may rotate each plane within the overlap without changing either the]*
11101 [failed]	*[\section{From the failed model to the typed algebra}]*
11147 [no]	*[operation; no null residue is returned.]*
11172 [No]	*[No member of \(\Sw\) is an absorber for every defined fusion.]*
11176 [not]	*[from a residue, and conditioning as a residue endomorphism are not]*
11199 [empty]	*[section is empty; the strict object language receives no empty]*
11203 [not]	*[\(c_{+}\circ c_{+}\) is not used when its codomain and domain do not]*
11241 [not]	*[algebra. It does not interpret the Riemann explicit formula, prove]*
11295 [not]	*[actions. They are not asserted to be literal models of every]*
11307 [not]	*[Chapter~\ref{ch:strict-grammar}; it does not depend on physically]*
11327 [no]	*[At four right angles the angular defect is zero, so no strictly]*
11328 [not]	*[convex positive-curvature vertex forms. This does not imply that]*
11331 [not]	*[interpretation of the bound is therefore an interpretation, not part]*
11364 [no]	*[through every axial plane. Horizontal position, which on the page carried no]*
11389 [not]	*[the flat notation could not absorb. For C_{3v} and $O(2)$ new classes]*
11391 [no]	*[(E, resp.\ E_m per angular wavenumber), pair-classes with no flat]*
11395 [not]	*[isotype, chiral pairs spanned the sign isotype. Folding does not remove]*
11435 [fails]	*[returns the flat page. The fourth right cone fails by blankness, not by]*
11472 [no]	*[recentering $y = x$ shifted to the parabola's axis (no origin required: the]*
11473 [not]	*[shift is a change, not a coordinate). The depressed equation reads]*
11492 [not]	*[the induced action of the resolvent factors through $S_4/V_4\cong S_3$; the root-stabilizer descent $S_4\supset S_3$ is the separate one-root picture,]*
11500 [not]	*[factor A_{5}: not cyclic, indeed simple, and isomorphic to the rotation]*
11506 [not]	*[same wall" is a structural correspondence, not a proved equivalence: what]*
11536 [not]	*[not apply.]*
11541 [not]	*[interpretation; it is not a derivation of the cubic formula from the]*
11558 [not]	*[stabilizer of one root. It is not the construction of the resolvent]*
11573 [not]	*[That reading is an interpretation of the classical theorem, not an]*
11579 [failed]	*[certificate, while the failed distinction is a metalinguistic]*
11580 [No]	*[nonseparation judgment. No Blank term is substituted for the]*
11615 [not]	*[\cite{cox}. The obstruction is not the prime five by itself:]*

line	*as it stands (diagnostic mention)*
11622 [No]	*[No general equivalence between physical folding constructions and]*
11638 [no]	*[meromorphic on a neighborhood of \(\overline\Omega\), with no pole on]*
11651 [not]	*[poles and their multiplicity records; it does not alter the classical]*
11657 [not]	*[function is not automatically an admissible counting kernel.]*
11729 [negative]	*[The sign depends on the variable. Statements that assign negative]*
11735 [no]	*[admitting no global labelled trivialization. Monodromy obstructs the]*
11736 [not]	*[latter, not the former.]*
11744 [no]	*[labels, so there is no global continuous enumeration preserving the]*
11752 [Neither]	*[which a trivialization is presented. Neither response licenses the]*
11791 [not]	*[its instances tabulated exactly — not as a theorem.]*
11799 [without]	*[without an orientation choice is the fixed component of the action:]*
11825 [no]	*[this volume and is asserted at no greater strength.]*
11839 [never]	*[a single root is never inscribable without exhibited orientation,]*
11849 [not]	*[The reading is a factorization schema, not a solvability]*
11852 [no]	*[distinct \(A_{5}\) obstruction for the generic quintic; no general]*
11855 [never]	*[dagger reading survives that boundary because it never crossed it:]*
11856 [not]	*[what is identified is the pattern of keeping, not any transfer of]*
11874 [no]	*[\section{The no-go and the two-circle barrier}]*
11884 [No]	*[No closed planar chain exists from one circle; the angle chain of any]*
11895 [not]	*[and the infimum is not attained. \emph{Proof.} $\sin$ is strictly concave]*
11901 [never]	*[2 is approached, never reached. \qed]*
11906 [no]	*[\emph{no} unconstrained minimizer: cheap closures degenerate. A minimizer]*
11933 [negative]	*[carries angle \emph{excess} (concentrated negative curvature). Closure and]*
11934 [no]	*[positive-deficit (single-circle) material are incompatible — the no-go of]*
11944 [No]	*[($\alpha_{\mathrm{off}}\approx 23.7^\circ$). No material is wasted: the]*
11976 [no]	*[$—n—\ge 1$ have $\nu_n\ge\nu_1=\sqrt{51}/6\approx1.19¿1$, limit-point, no data]*
11986 [no]	*[$\arcsin(1/\sqrt5)\approx 26.57^\circ$ have \emph{no} apex freedom at all.]*
12002 [absent]	*[chain thus supports a genuinely new observable absent from open chains:]*
12066 [no]	*[Dirichlet decoupling $U=-I$ is the disjoint case (no junction; balances blank);]*
12071 [not]	*[the stated items; not a formal equivalence.}]*
12087 [not]	*[material (Prop.~\ref{prop:saddle}), the unconstrained minimum does not]*
12088 [no]	*[exist (Prop.~\ref{prop:inf}), the symmetric minimizer wastes no element]*
12130 [not]	*[The presence interpretation does not interpret \(\xi\) as a total]*
12131 [null]	*[map into a field containing a null value. It retypes the classical]*
12144 [no]	*[If the completed evaluation procedure finishes at \(s\) but emits no]*
12152 [not]	*[It is not an element of \(\Ctimes\), a third truth value, or an]*
12177 [no]	*[\label{prop:no-tally-identity}]*
12190 [no]	*[Thus no \(n\in T\) satisfies \(m\oplus n=m\) for every \(m\).]*
12205 [No]	*[Definition~\ref{def:strict-terms}. No zero atom occurs, and no]*
12210 [not]	*[This is a theorem about \(\GP\). It is not a claim that]*
12254 [no]	*[symmetric balance relation. It requires no Blank control and no]*
12336 [not]	*[Sign-change detection is therefore not a fourth unconditional]*
12347 [negative]	*[Thus the negative even zeros belong to the uncompleted zeta]*

line	*as it stands (diagnostic mention)*
12387 [negative]	*[negative-sector observation — not a record of failed formation.]*
12399 [negative]	*[and the negative-sector counter-witness at \((D,s)\) holds exactly]*
12430 [not]	*[It is not defined as a positivity criterion. Distinctly, the]*
12458 [not]	*[\(\CounterAdmClass\) is not thereby a model of the zeta reference]*
12497 [not]	*[zeta-specific Admission law. It is not a derivation of that law from]*
12498 [not]	*[the unextended base theory, and it is not advertised as an unpriced]*
12529 [not]	*[does not establish them.]*
12592 [not]	*[criterion is a classical theorem used at the interface, not a]*
12623 [negative]	*[\(s-\tfrac12\) to its negative, so it fixes]*
12641 [not]	*[whose real part is \(4/5\), not \(1/2\). The roots form a complete]*
12648 [not]	*[information not shared by \(F\), such as the prime genealogy,]*
12660 [fails]	*[symmetry bookkeeping. The route fails, and the failure is exact:]*
12674 [not]	*[\(\Pbase\), not PA, ZFC, or another external proof theory.]*
12693 [no]	*[\item no class-wide Admission axiom.]*
12821 [not]	*[sentences. The literal classical zero-locus sentence is not in]*
12823 [not]	*[sentence. The theorem also does not establish independence of]*
12834 [not]	*[does not by itself verify:]*
12844 [No]	*[No script execution is asserted by this TeX source.]*
12851 [negative]	*[Fix a decidable address sort and a decidable negative verifier. Write]*
12892 [not]	*[They are not strict object-language numerals and do not install a]*
12904 [not]	*[The rule does not infer a family from]*
12931 [no]	*[events. They contain no rule whose premise is merely]*
12934 [without]	*[therefore leaves the stagewise statement without a family-forming]*
12936 [not]	*[not a theorem that no external classical choice principle can be]*
12950 [not]	*[This is a completion postulate. It does not provide:]*
12966 [negative]	*[accepted by the negative verifier; and a kept stage covering that]*
12990 [not]	*[This is not an analytic proof because the completed-family premise is]*
12995 [not]	*[The two-model theorem concerns \(\AdmClass\), not]*
12997 [not]	*[base does not select a side of the class-wide fork.]*
13013 [negation]	*[\item the proof that its negation is a finite-witness]*
13056 [not]	*[theorem internal to the standard analytic interface, not a bound on]*
13062 [no]	*[coverage interface \(C_{\mathrm{cov}}\) defined below, and no clause]*
13070 [without]	*[other's without the corresponding transfer theorem; the asymmetry]*
13090 [no]	*[\item has no representative in the standard countable analytic]*
13100 [fails]	*[If \(C_{\mathrm{cov}}\) fails, truth of the arithmetic shadow does not]*
13110 [Without]	*[Without coverage, a formation outside the image is not addressed by]*
13111 [No]	*[the arithmetic quantifier. No transfer follows for that formation.]*
13117 [not]	*[program does not silently promote that internal theorem into a proof]*
13120 [not]	*[the content of \(C_{\mathrm{cov}}\), not a free consequence of]*
13158 [not]	*[therefore not an evasion of the Riemann question: at the]*
13163 [not]	*[by an instance-separation theorem that the reference clauses do not]*
13168 [No]	*[No classical analytic system can simultaneously assert that it]*

line	*as it stands (diagnostic mention)*
13169 [fails]	*[faithfully realizes \(\Pdag\), that zeta-specific Admission fails,]*
13172 [failure]	*[failure of \(\RHClass\) demonstrates failure of the realization's]*
13173 [not]	*[faithfulness, not of the native theorem.]*
13179 [not]	*[not refer to \(\zeta\); Model˜B separates the class-wide fork]*
13183 [cannot]	*[descent cannot be internalized as a base refutation, because the]*
13184 [not]	*[base does not name the object the zero would live in. The entire]*
13186 [without]	*[relocated, without remainder, into one construction problem —]*
13202 [not]	*[than arbitrary. The answer is not an analogy to any historical]*
13209 [not]	*[witness-not-found, no counterexample-not-seen, no checker-returned-nothing.]*
13215 [negative]	*[selected package of the completed zeta descent: exact finite negative]*
13219 [empty]	*[identically empty \cite{emmerson-witness-fibres}. Equivalent criteria are]*
13221 [not]	*[availability of an exact criterion is not occupation of a sign. Criterion]*
13223 [without]	*[verification raise complexity without raising sign]*
13243 [without]	*[The license follows. A route that forms a sign without exposing its]*
13245 [no]	*[which admits no untethered premise into a sign derivation. A route whose formation withholds every inscription is already absorbed]*
13247 [not]	*[therefore not one option among many equally available foundational]*
13271 [not]	*[is accordingly the instrument of reach, not a limit on it. The theorem]*
13272 [no]	*[is a proved reaching, and it grips \(\RHClass\) exactly. It requires no]*
13275 [not]	*[realizations of \(\Pdag\), and is not a hypothesis of the interface]*
13284 [not]	*[That trace is not a qualification on the verdict. It is what a verdict]*
13369 [Without]	*[On the Riemann hypothesis these parameters are real. Without that]*
13401 [not]	*[classical formula. The phrase "four unsigned blocks" is not used.]*
13408 [not]	*[arithmetic. Those evaluations are not analytic tail certificates.]*
13429 [No]	*[No theorem in the assertion-of-record stratum uses the former]*
13440 [not]	*[table is retained as numerical data at the stated truncations, not as]*
13501 [not]	*[multiprecision arithmetic. This TeX source does not claim that those]*
13505 [never]	*[Because a Gaussian has full support, its prime bill is never]*
13506 [absent]	*[grammatically absent merely because it is numerically small. The]*
13507 [empty]	*[exact empty-bill statement belongs only to a compactly supported test]*
13523 [no]	*[has no contributing term.]*
13534 [absence]	*[The absence of a contributing term is stated here in the]*
13536 [not]	*[diagnostic, but that display is not a strict empty object.]*
13551 [not]	*[observations, not a completed proof of the global tail condition.]*
13556 [absence]	*[specified off-axis quartet, but absence of observed oscillation over]*
13557 [not]	*[a finite range does not exclude all off-axis configurations.]*
13560 [not]	*[Detector is therefore exploratory and partial. It is not a]*
13563 [No]	*[\section{No analytic-budget theorem}]*
13569 [not]	*[are not assertions of record. They require a repaired zero-count]*

line	*as it stands (diagnostic mention)*
13622 [not]	*[A law is an act performed at every stage, not a stasis — the idiom]*
13655 [absence]	*[chapter summarized the survival roster with an absence idiom]*
13740 [not]	*[sufficiency is not a theorem of this volume. One external trial is]*
13744 [not]	*[name; a trial supports and does not prove.]*
13754 [no]	*[Chapter~\ref{ch:explicit-formula}, and no positivity theorem is]*
13785 [neither]	*[roster below the cited height, and neither is asserted here.]*
13792 [fails]	*[positivity structure fails are of distinct genealogy and prime-fused]*
13794 [no]	*[lies inside the domain of Native Admission, and no discharge of]*
13815 [neither]	*[so the class-wide hypothesis itself is neither asserted nor refuted]*
13818 [fails]	*[that every known off-axis family fails a filter is recorded at]*
13832 [No]	*[Davenport–Heilbronn exclusion. No run is claimed.]*
13844 [no]	*[which \(\Phi¿0\) on a right neighborhood of the line, or show no]*
13854 [not]	*[explicit-formula ledger. (7)~Construct an admitting model of the referenced theory \(\Pbase+\mathrm{RefClauses}\) — a model of the reference clause]*
13867 [not]	*[\(\AdmClass\), not \(\AdmZeta\), the full coverage interface, or an]*
13969 [No]	*[No Admission sentence is built into the base signature.]*
13985 [not]	*[renderer-control positions do not become strict semantic operands\\]*
14001 [no]	*[no absorber among proper sweeps\\]*
14030 [not]	*[axioms below are satisfied. A checker run or Lean theorem is not]*
14176 [fails]	*[fails at \(s_0\) by the exact fraction\\]*
14194 [No]	*[No zeta-specific or higher-cardinality coverage conclusion follows]*
14195 [without]	*[without another interface theorem.]*
14205 [not]	*[distribution is the separate \texttt{.lean} file, not this typeset]*
14209 [no]	*[\texttt{blank}, for which the strict discipline of record has no]*
14214 [No]	*[No compiler result or checksum is asserted by this transcription.]*
14221 [not]	*[It is not the complete strict PreTerm/Term split. -/]*
14590 [no]	*[the discipline of record carries no non-emission mark at any layer;]*
14613 [not]	*[orbit spans decompose isotypically and need not be irreducible\\]*
14617 [not]	*[zero angular defect does not exclude every nontrivial flat fold\\]*
14633 [missing]	*[the claimed derivations used an unsound or missing representation\\]*
14658 [without]	*[the sponsor forms the completed family without supplying computable]*
14663 [not]	*[the zero-count and outward-rounded tail work was not completed in the]*
14668 [not]	*[contains implementation \texttt{blank} and does not formalize the]*
15286 [not]	*[The mathematical source does not hard-code a compiler result,]*

Site prose

60 transitions across the homepage, Four Statements, Symmetry, Glossary, Open Problems, The Seven, and Future Directions.

line	*as it stood*	*in presence*
shell pages	*[the numeral zero does not occur]*	every numeral is formed by strokes [the numeral zero stands outside the inventory]
index.html	*[Why symmetry alone is not enough]*	Why the content lies beyond symmetry
index.html	*[You may never pass.]*	Every turn is an act.
index.html	*[No bluffing — only things not yet done.]*	Every claim on the table is enacted or standing open — each in its own name.
index.html	*[Cost is weight, not price.]*	Cost is weight, carried on the face.
index.html	*[Three columns, never summed.]*	Three columns, each counted under its own name.
index.html	*[Finish never changes gameplay.]*	Finish is finish; play is play.
index.html	*[without being taught a word of it.]*	ahead of being taught a word of it.
index.html	*[holds the checker in full and no answer.]*	holds the checker in full; the answer stays with you.
statements.html	*[the withheld value is not made into an object.]*	the withheld value stays a metalanguage judgment.
shell pages	*[Why symmetry is not enough]*	Why the content lies beyond symmetry
shell pages	*[It cannot: there is an explicit polynomial]*	The witness settles it: there is an explicit polynomial
shell pages	*[Why Symmetry Alone Cannot Prove the Riemann Hypothesis]*	Why the Riemann Hypothesis Lies Beyond Symmetry Alone
symmetry.html	*[The failure is worth understanding precisely:]*	The route's exact stopping point is worth understanding precisely:
glossary.html	*[the numeral zero does not occu]*	every numeral is formed by strokes [the numeral zero stands outside the inventory]; and the sign occu
index.html	*[Non-presentation forms no substitute term.]*	Every term has a sponsor, and a sponsor is a presentation.
glossary.html	*[the numeral zero, the empty collection, the null return, default falsity, the vacuous judgment, a negative rec]*	the numeral zero and its surrogates — [the empty collection, the null return, default falsity, the vacuous judgment, a negative rec
shell pages	*[cost is weight, not price]*	cost is weight, on the face
glossary.html	*[the withheld value is not converted into an object.]*	the withheld value stays a metalanguage judgment.
shell pages	*[Content expressible without an orientation choice]*	Content expressible ahead of any orientation choice
shell pages	*[or prove that the referenced theory does not derive]*	or prove that the referenced theory leaves underived
open-problems.html	*[so the proved class-wide theorem neither suppl]*	so the proved class-wide theorem stands apart: it suppl
shell pages	*[the checker that holds no answer]*	the checker; the answers stay with you
the-seven.html	*[holds the checker in full and the answer in no form at all.]*	holds the checker in full; the answer stays with you, in every form.
the-seven.html	*[plays without it;]*	plays free of it;
the-seven.html	*[is the one thing the sandbox does not contain.]*	is the one thing that stays outside the sandbox.
shell pages	*[a lock is not opened until its worked example checks]*	a lock opens exactly when its worked example checks
future-directions.html	*[it could not add a third place to put asymmetric content.]*	a third place for asymmetric content lay beyond it.
future-directions.html	*[That is not the volume's result and it is not right.]*	The volume's result stands elsewhere, and stands corrected here.

line	*as it stood*	*in presence*
future-directions.html	*[The dual OBox never had: open below.]*	The dual OBox stood to gain: open below.
future-directions.html	*[UBox leaves its ORIGIN so — which is what an antiderivative has: no distingu]*	UBox leaves its ORIGIN so — which is what an antiderivative has: its distinguished origin standing open; a distingu
future-directions.html	*[Chirality is never carried by a glyph in this notation, on any carrier.]*	Every glyph of this notation is fixed under reflection, on every carrier.
future-directions.html	*[carries no chiral glyph anywhere.]*	carries reflection-fixed glyphs throughout.
future-directions.html	*[though the direction of the trade is not.]*	with the direction of the trade standing open.
shell pages	*[the withheld value is not made into an object]*	the withheld value stays a metalanguage judgment
shell pages	*[Why symmetry is not enough]*	Why the content lies beyond symmetry
shell pages	*[It cannot: there is an explicit polynomial]*	The witness settles it: there is an explicit polynomial
shell pages	*[the numeral zero does not occur]*	every numeral is formed by strokes [the numeral zero stands outside the inventory]
shell pages	*[Non-presentation forms no substitute term]*	Every term has a sponsor, and a sponsor is a presentation
shell pages	*[the numeral zero, the empty collection, the null return, default falsity, the vacuous judgment, a negative record]*	the numeral zero and its surrogates — [the empty collection, the null return, default falsity, the vacuous judgment, a negative record
shell pages	*[cost is weight, not price]*	cost is weight, on the face
shell pages	*[the withheld value is not converted into an object]*	the withheld value stays a metalanguage judgment
shell pages	*[Content expressible without an orientation choice]*	Content expressible ahead of any orientation choice
shell pages	*[or prove that the referenced theory does not derive]*	or prove that the referenced theory leaves underived
shell pages	*[the checker that holds no answer]*	the checker; the answers stay with you
shell pages	*[holds the checker in full and the answer in no form at all]*	holds the checker in full; the answer stays with you, in every form
shell pages	*[plays without it]*	plays free of it
shell pages	*[is the one thing the sandbox does not contain]*	is the one thing that stays outside the sandbox
shell pages	*[a lock is not opened until its worked example checks]*	a lock opens exactly when its worked example checks
shell pages	*[Why Symmetry Alone Cannot Prove the Riemann Hypothesis]*	Why the Riemann Hypothesis Lies Beyond Symmetry Alone
shell pages	*[The failure is worth understanding precisely]*	The route's exact stopping point is worth understanding precisely
shell pages	*[so the proved class-wide theorem neither supplies]*	so the proved class-wide theorem stands apart: it leaves unsupplied
future-directions	*[it could not add a third place to put asymmetric content]*	a third place for asymmetric content lay beyond it
future-directions	*[It passed on atoms and failed on precisely those]*	It passed on atoms and stopped exactly at those
future-directions	*[Chirality is never carried by a glyph in this notation, on any carrier]*	Every glyph of this notation is fixed under reflection, on every carrier

line	*as it stood*	*in presence*
future-directions	*[carries no chiral glyph anywhere]*	carries reflection-fixed glyphs throughout
future-directions	*[the direction of the trade is not]*	the direction of the trade stands open
index.html	*[Non-presentation forms no substitute term.]*	Every term has a sponsor, and a sponsor is a presentation.
index.html	*[No majority is a]*	A split table is a
shell pages	*[is ¡em¿not¡/em¿ an\s*unconditional equivalent]*	stands ¡em¿beside¡/em¿ the three unconditional equivalents

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The renderer, checker, generated plates, Lean sources, exact-arithmetic checker, numerical scripts, notebook records, and build manifest belong to the publication bundle.

Language-model instruments were used under the author's direction for drafting, restructuring, LaTeX work, software assistance and technical review [1, 2]; they are cited as sources. Every definition, adopted law, theorem statement, correction, interface and publication claim is the author's.

The mathematical source does not hard-code a compiler result, execution result, short hash, full artifact hash, or archive hash. The final distribution's `README.md`, execution logs, and `build-manifest.json` are authoritative for those facts after the archive is frozen.

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